

Modeling and simulation of neuronal morphology

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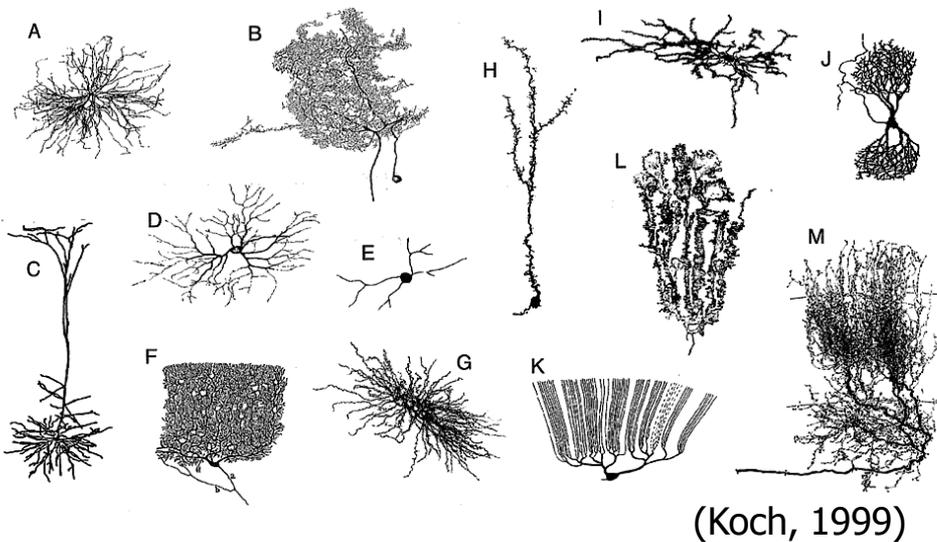
ATR Neural Information Analysis Laboratories



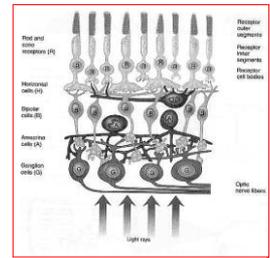
Morphological divergence of neurons

Basic hypothesis:

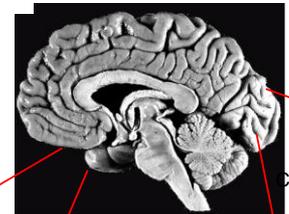
Morphology and structure are **information representation** in the brain.



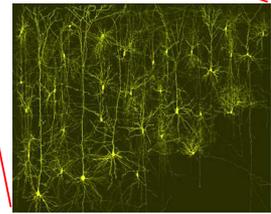
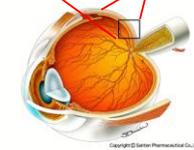
(Koch, 1999)



Ganglion, Bipolar, Rod,...



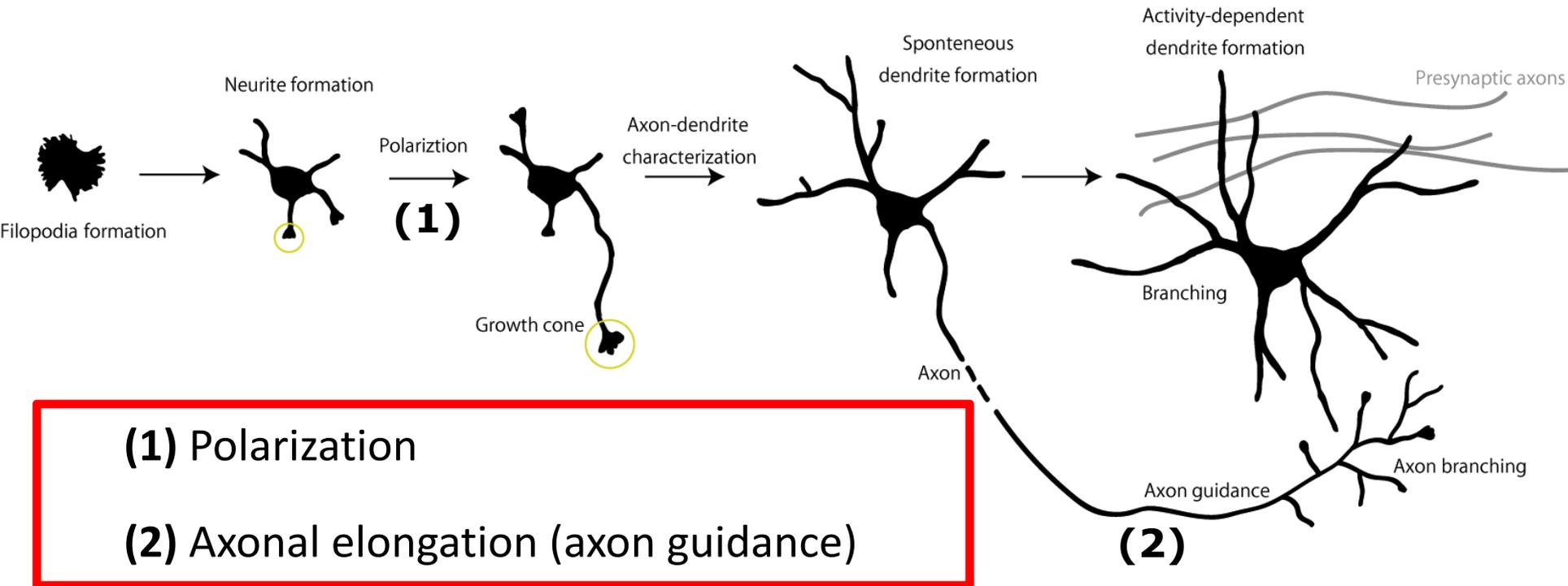
Cortical pyramidal neurons



Development process in the brain

How do neurons obtain their morphology?
How do neural circuits form their patterns?

Structural plasticity in neural development

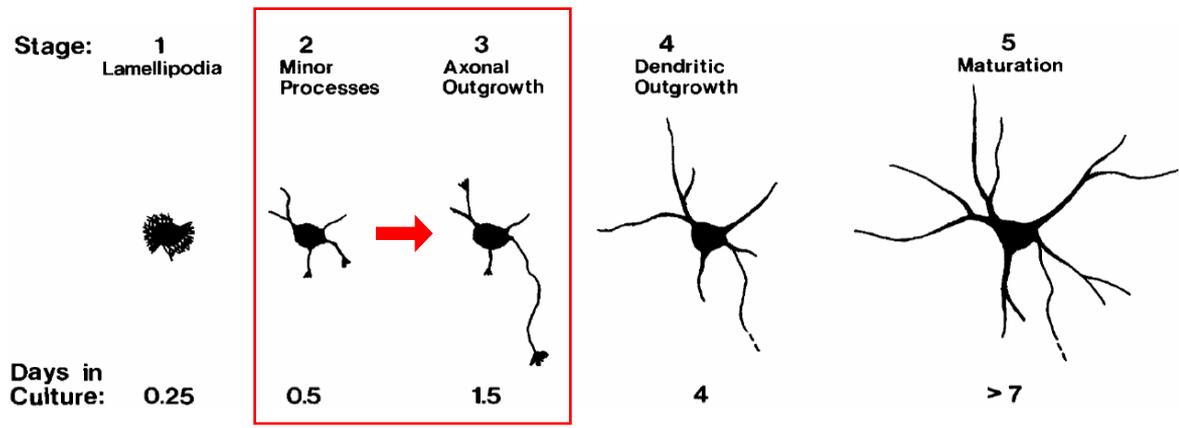


- A decoding process from analog (and sometimes weak) molecular signals to digital morphology
- Can be symmetry breaking phenomena

Spontaneous neuronal polarization: model, mathematics and biology

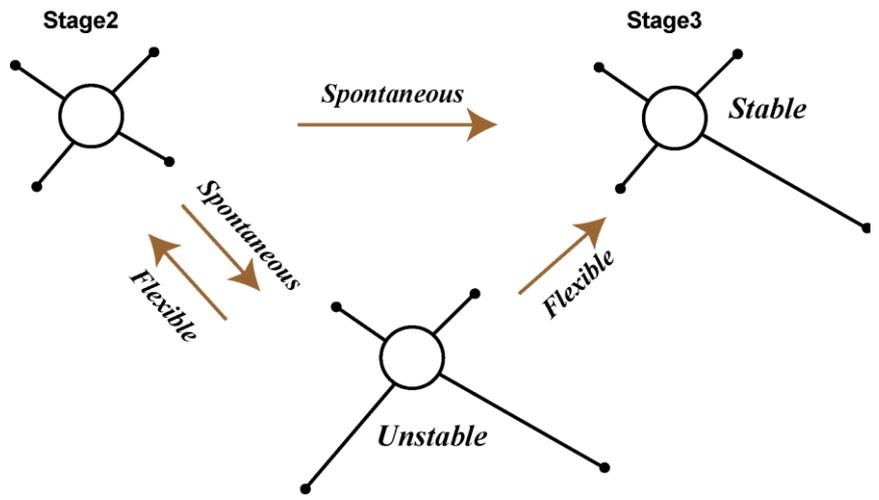
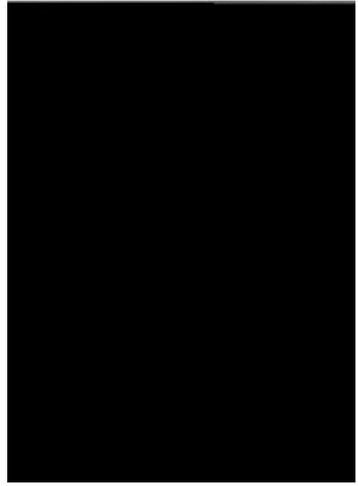
Naoki, H., Nakamuta, S., Kaibuchi, K., Ishii, S. PLoS ONE, 6, 2011.
Toriyama, M., Sakumura, Y., Shimada, T., Ishii, S., Inagaki, N.
Molecular Systems Biology, 6, 2010.

Axon determination of differentiated neurons



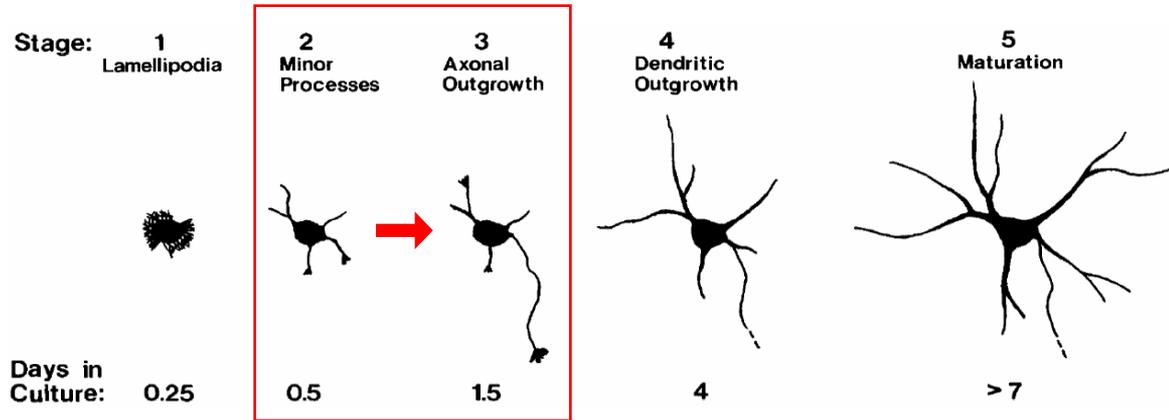
Dotti et. al, J Neurosci, 1988

from website of Banker Lab



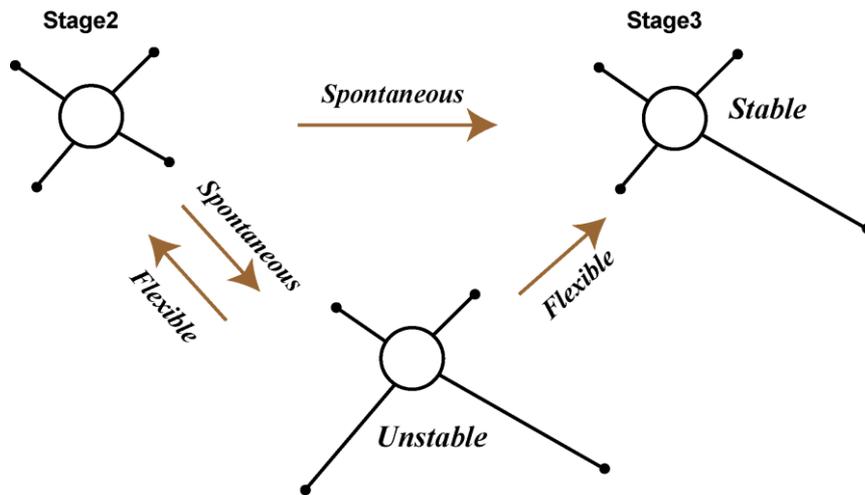
- **Spontaneity**
A neuron is *spontaneously* polarized, even in a uniform extracellular condition.

Axon determination of differentiated neurons



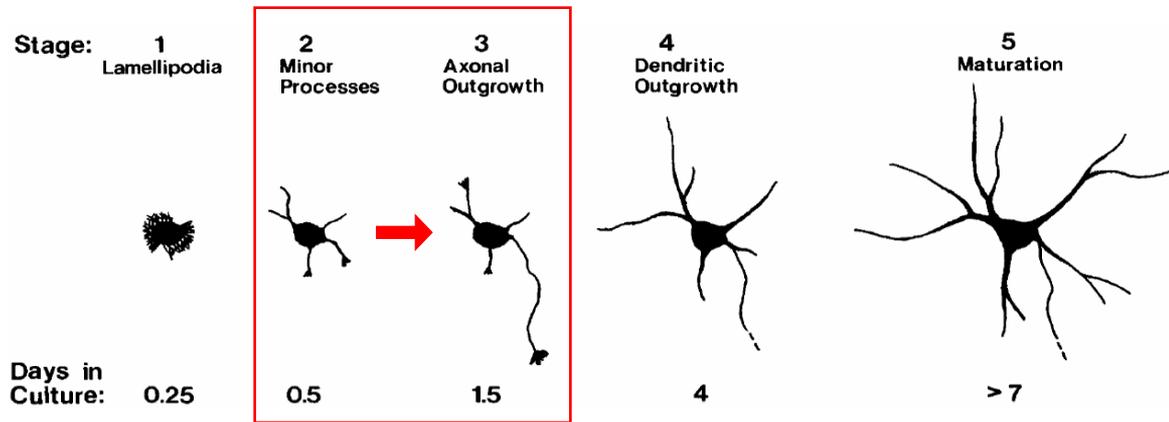
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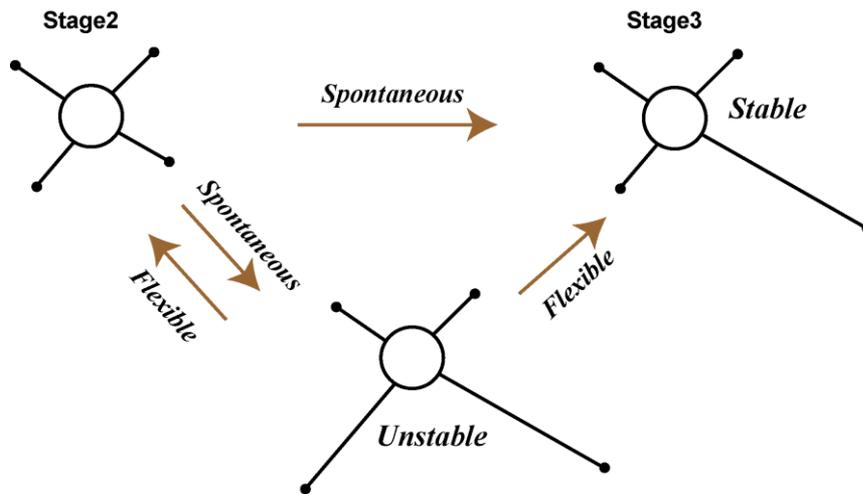
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- **Stabilization**
Once a single axon is selected, remaining neurites cannot be elongated.

Axon determination of differentiated neurons



Dotti et. al, J Neurosci, 1988

from website of Banker Lab



- **Spontaneity**
A neuron is *spontaneously* polarized, even in an uniform extracellular condition.
- **Stabilization**
Once a single axon is selected, remaining neurites cannot be elongated.
- **Correction**
Sometime, multiple neurites mistakenly happen to be selected, but this failed pattern is *flexibly* cancelled out to yield a single axon.

What is the mechanism of such a flexible morphogenesis?

Model: axon determination molecule

- **Compartment model**
 - **Soma, several neurites**
- Axon determination molecule: factor X
 - Gene expression in the soma
 - Degradation or inactivation
 - Diffusion
 - Active transport from the soma to each neurite tip

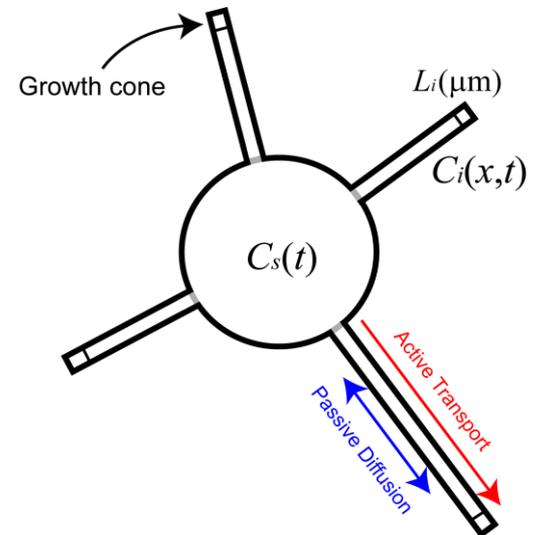
ODE (Concentration of factor X **in the soma**)

$$\frac{dC_s}{dt} = G - kC_s + \frac{1}{V} \sum_i^N \left[DA \frac{\partial C_i(0,t)}{\partial x} - \alpha C_s \right]$$

PDE (Concentration of factor X **along each neurite**)

$$\frac{\partial C_i}{\partial t} = D \frac{\partial^2 C_i}{\partial x^2} - kC_i - D \frac{\partial C_i(0,t)}{\partial x} \delta(0) + \frac{\alpha C_s}{A} \delta(L_i)$$

Neuroinformatics 2011



Model: axon determination molecule

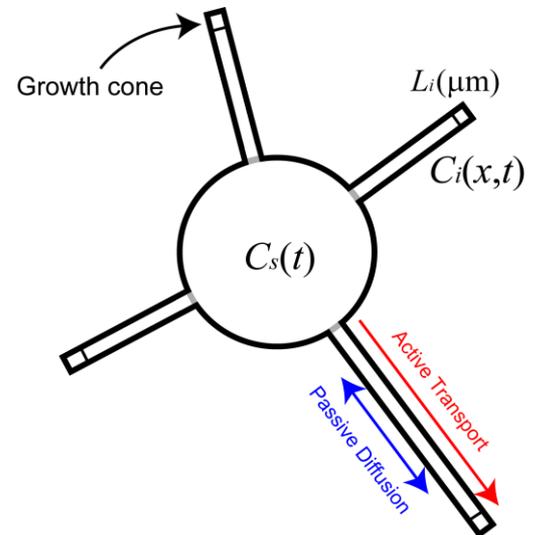
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Model: axon determination molecule

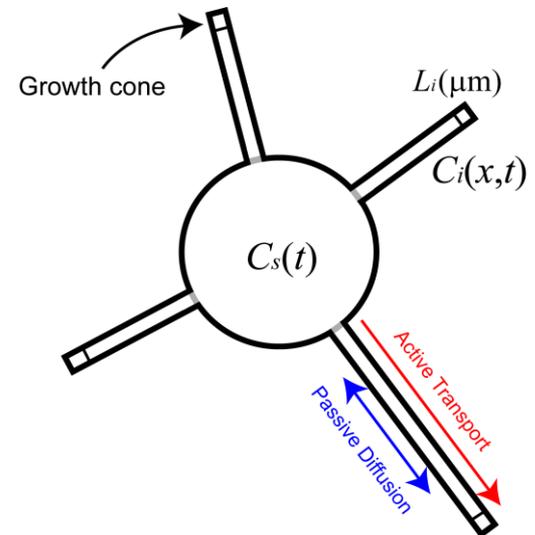
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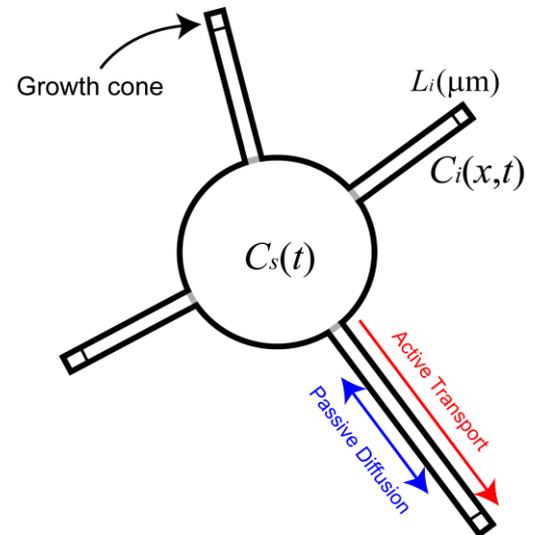
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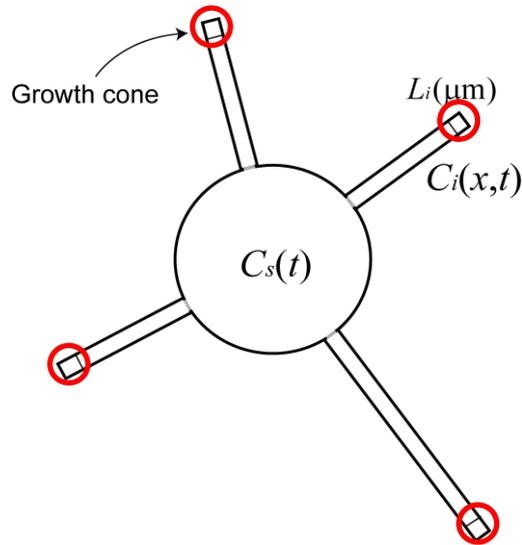
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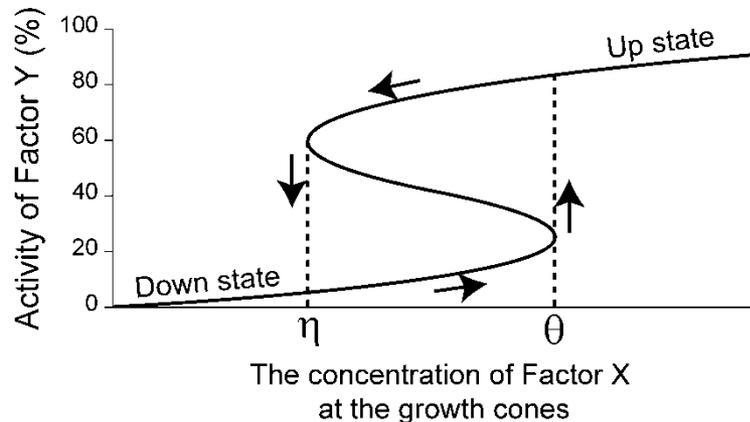
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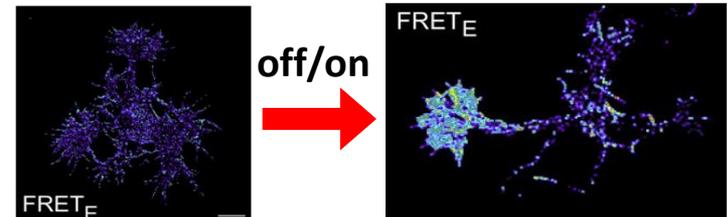
Model: cytoskeleton regulating molecule



- Cytoskeleton regulating molecule: factor Y
 - **Work at each neurite tip**
 - Activated by the axon determination factor (X)
 - Bistable switch (hysteresis)



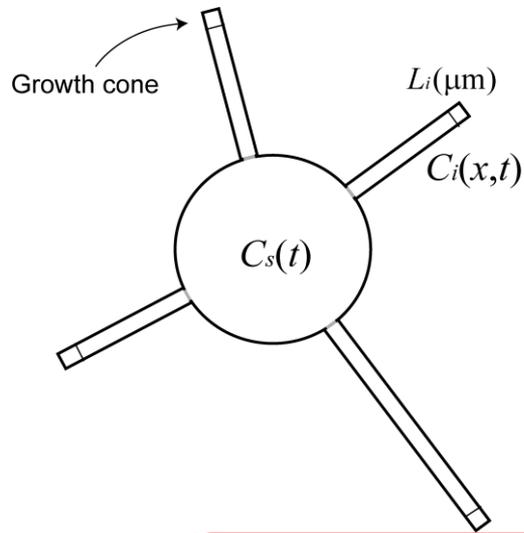
- When factor Y is "up state", neurite elongates.
- When factor Y is "down state", neurite shrinks.



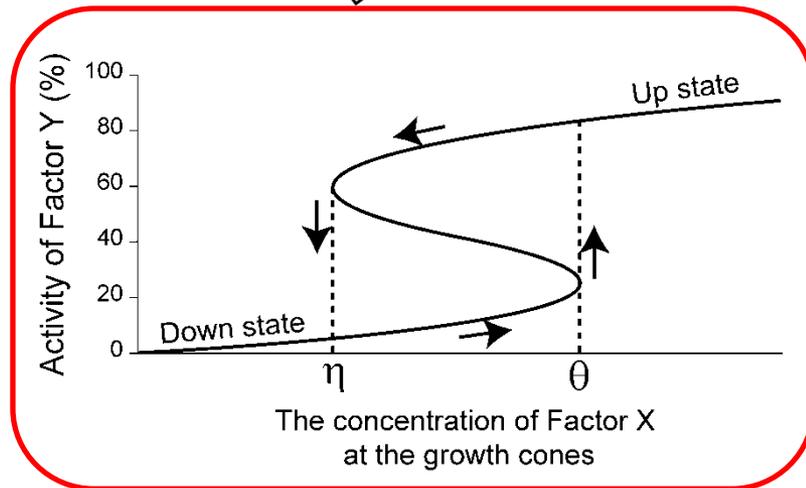
Fivaz et. al, Curr Biol, 2008

HRas behaves like a molecular switch, being highly activated in the selected neurite.

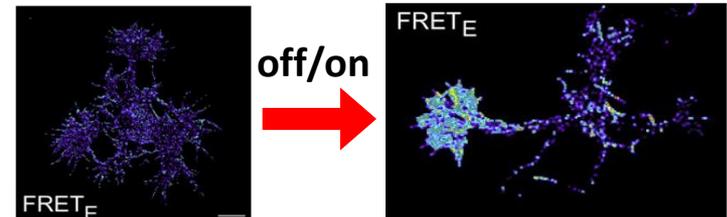
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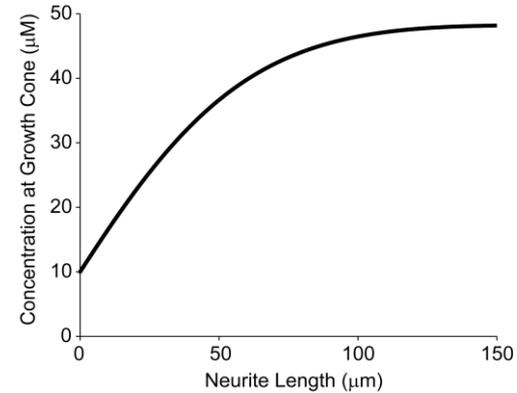


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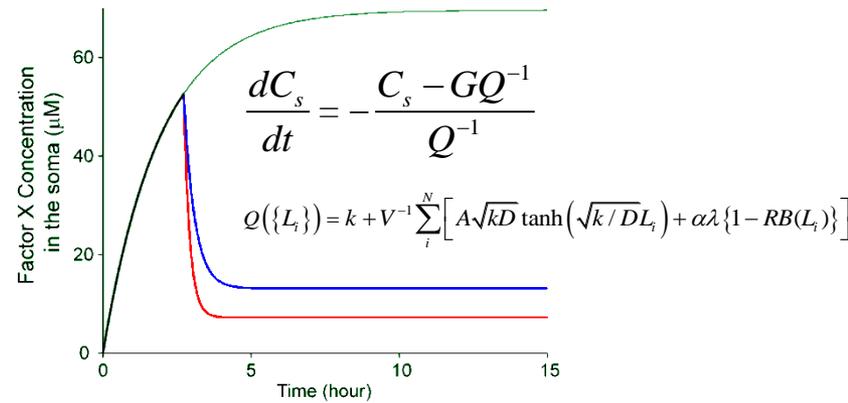
Mathematics: Winner-take-all mechanism

- **Local activation** in the growth cone
 - Factor X is accumulated in the growth cone due to the increasing active transportation
- **Global inhibition** in the somatic pool
 - As the neurite becomes long, it is difficult for factor X to diffuse back to the soma.
 - If there are long neurites, the somatic pool of factor X is dried up.

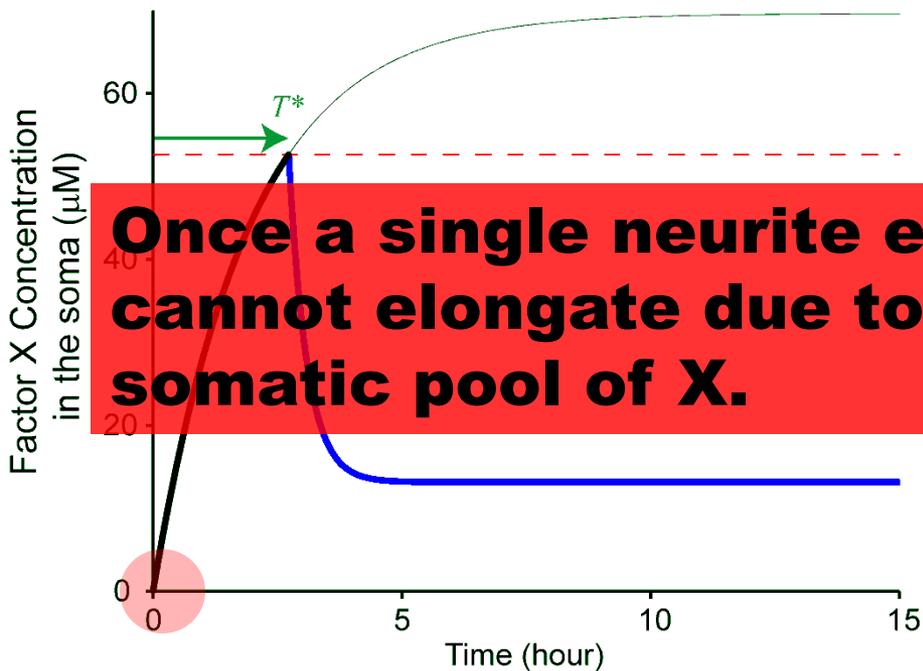
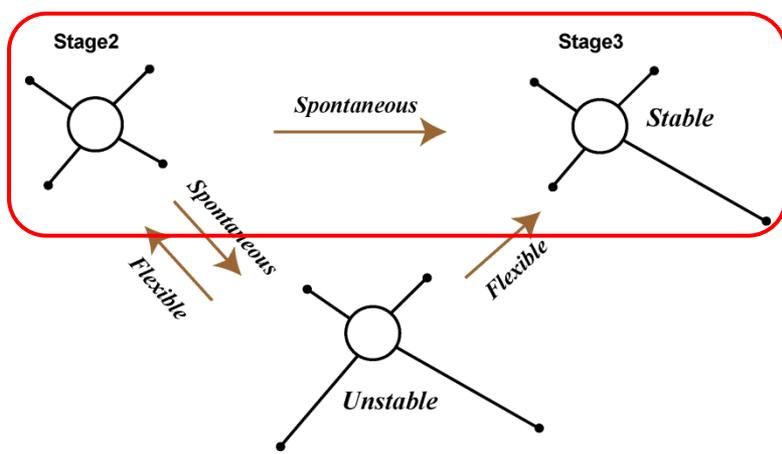


$$C_i(L_i) = C_s \left[\frac{\alpha}{A\sqrt{kD}} \tanh(\sqrt{k/D}L_i) + \cosh(\sqrt{k/D}L_i)^{-1} \right]$$

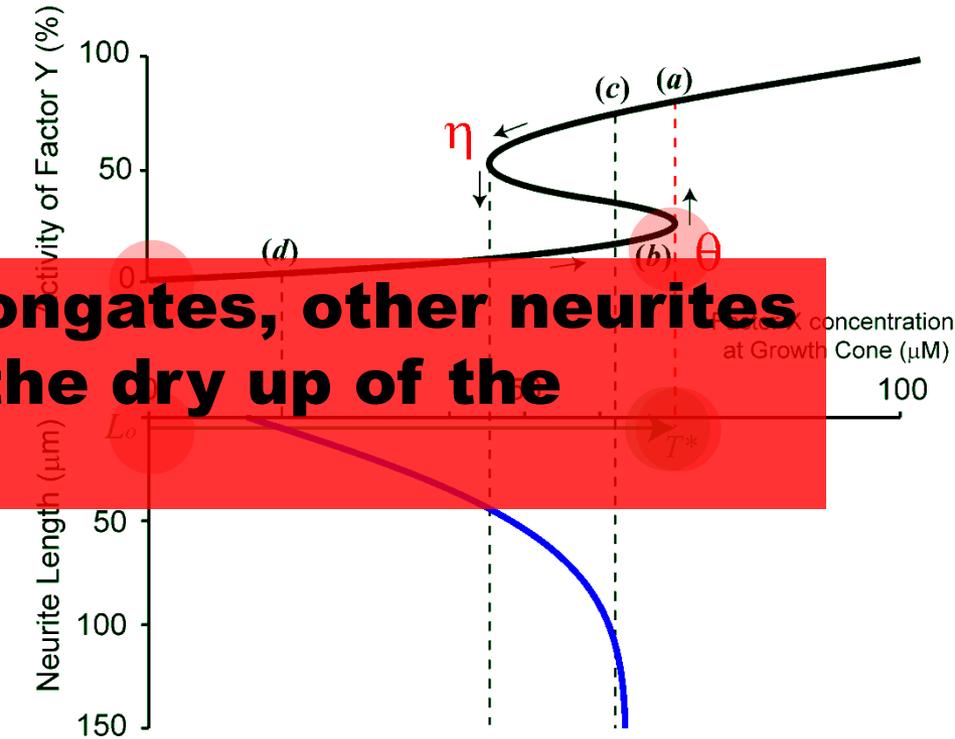
No long neurite
 Single long neurite
 Two long neurites



Mathematics: Spontaneity and stability



Activity of factor Y (depending on factor X)

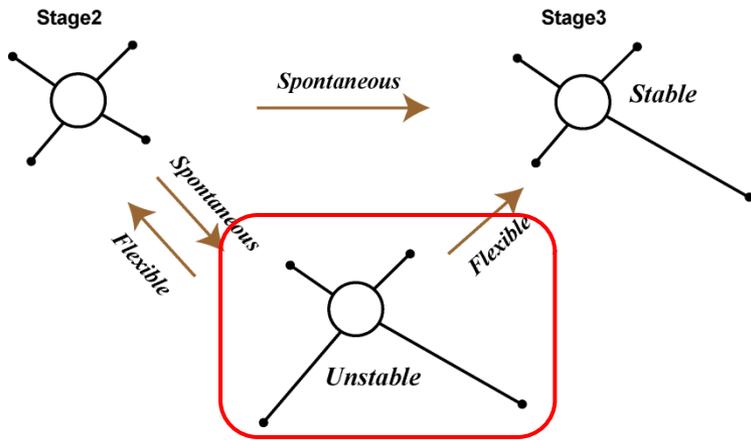


Once a single neurite elongates, other neurites cannot elongate due to the dry up of the somatic pool of X.

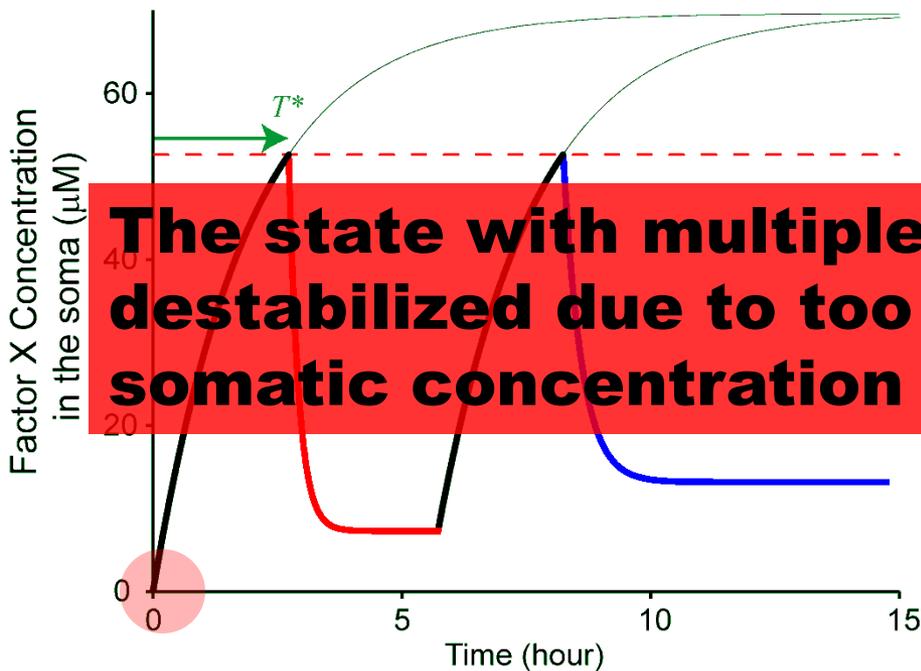
Dynamics of factor X in the soma

Concentration of factor X in the neurite tip
vs Neurite length

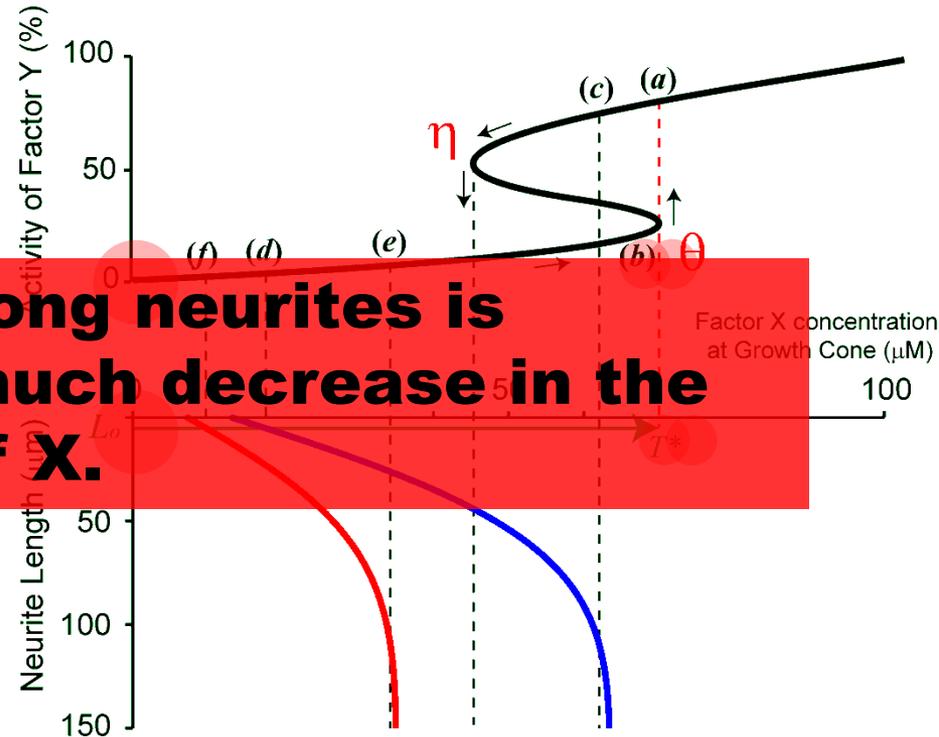
Mathematics: Correction



Activity of factor Y (depending on factor X)



The state with multiple long neurites is destabilized due to too much decrease in the somatic concentration of X.

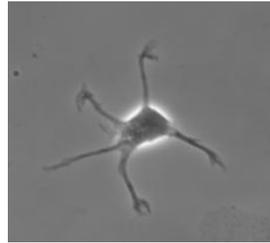


Dynamics of factor X in the soma

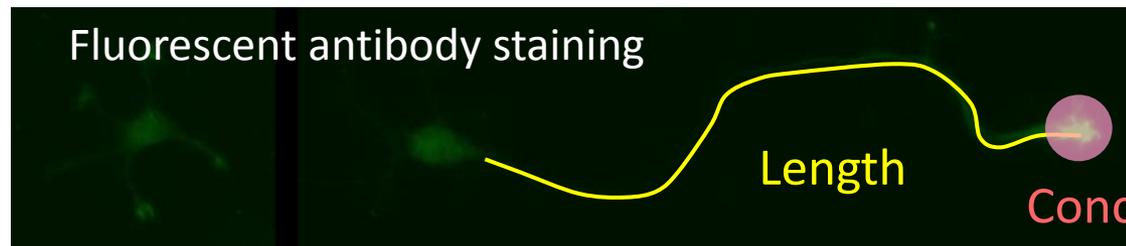
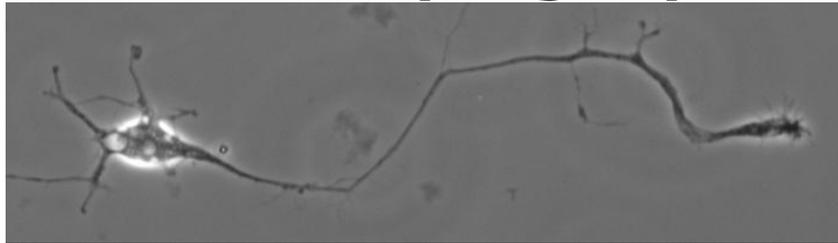
Concentration of factor X in the neurite tip
vs Neurite length

Biology: Neuronal polarization correlates with expression of Shootin1

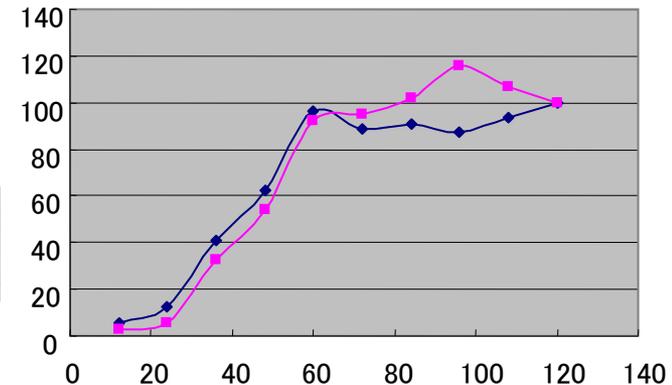
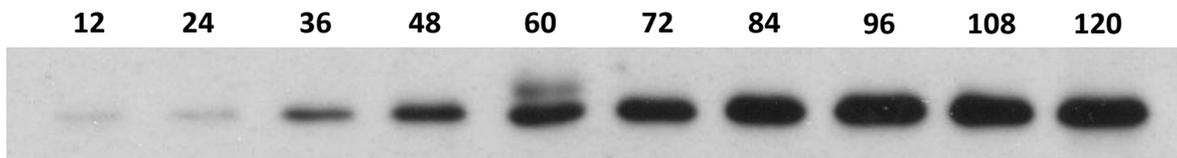
**Before polarization
(Stage 2)**



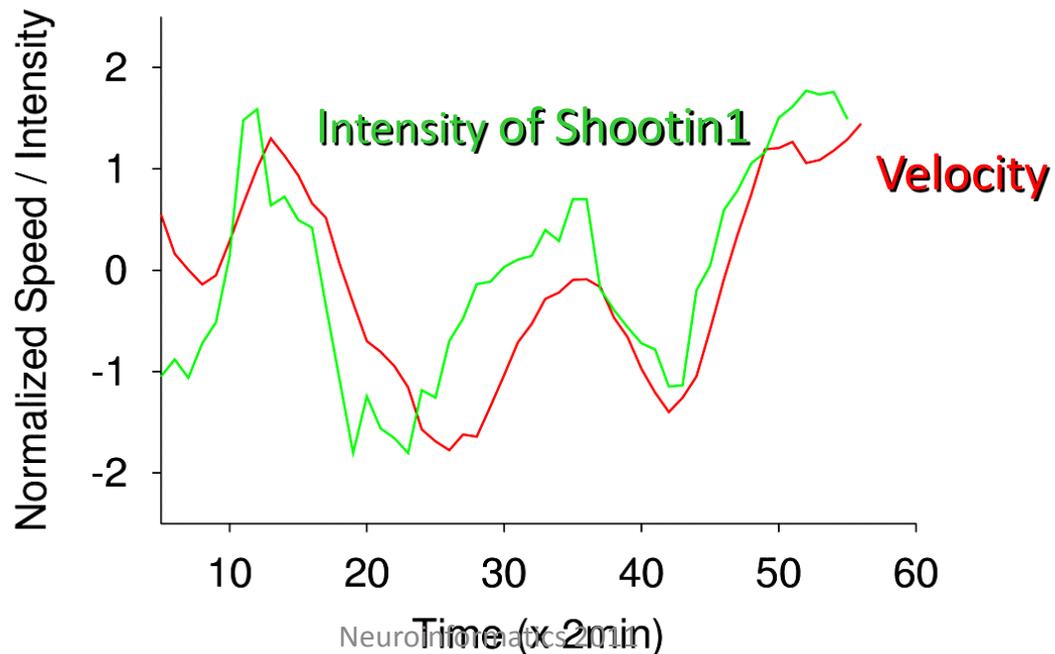
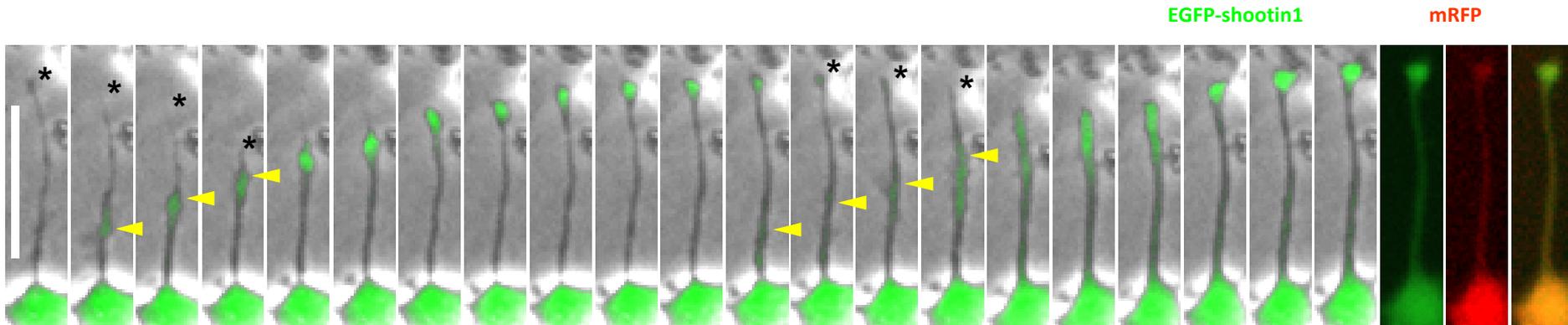
**After polarization
(Stage 3)**



Expression rate of Shootin1

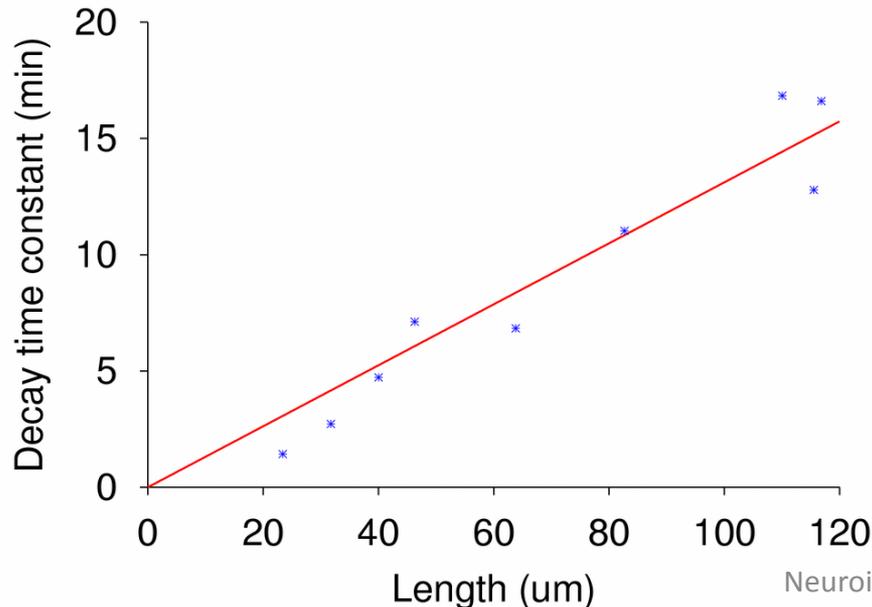
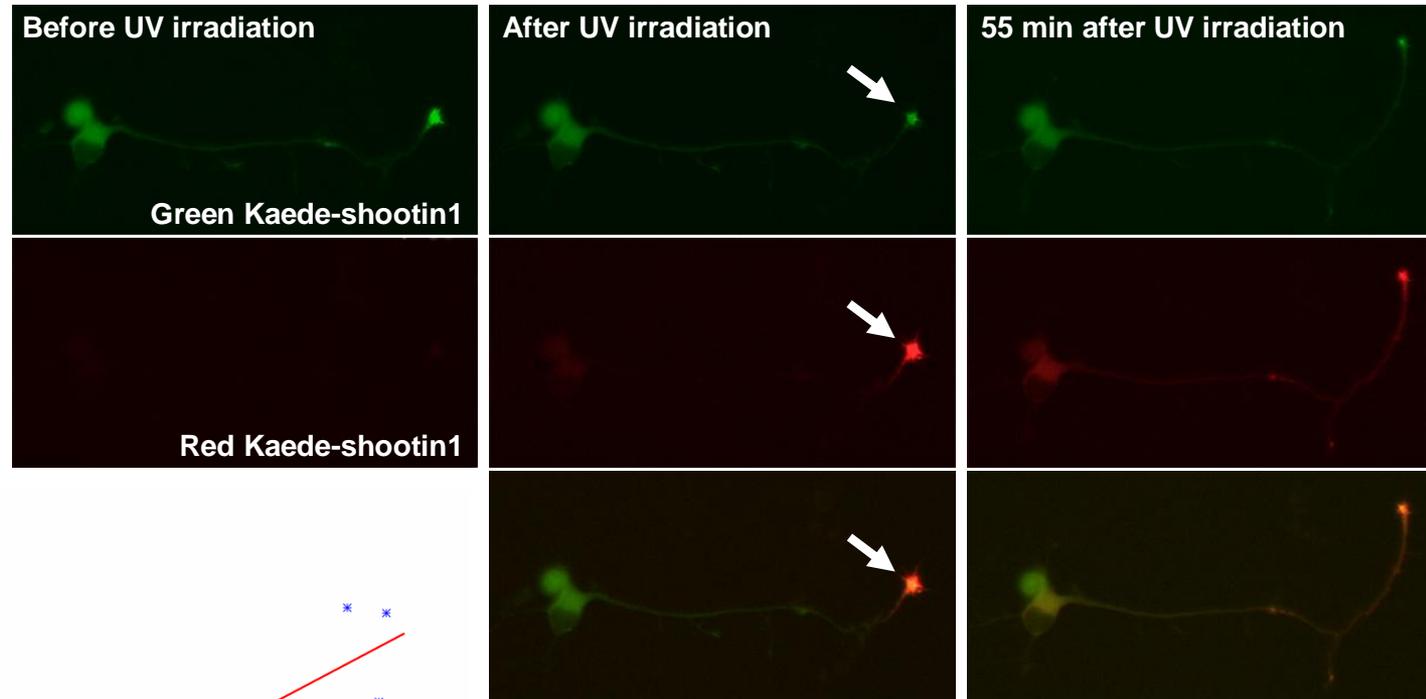


Biology: Active transportation of Shootin1



Biology: Estimation of the diffusion constant of Shootin1

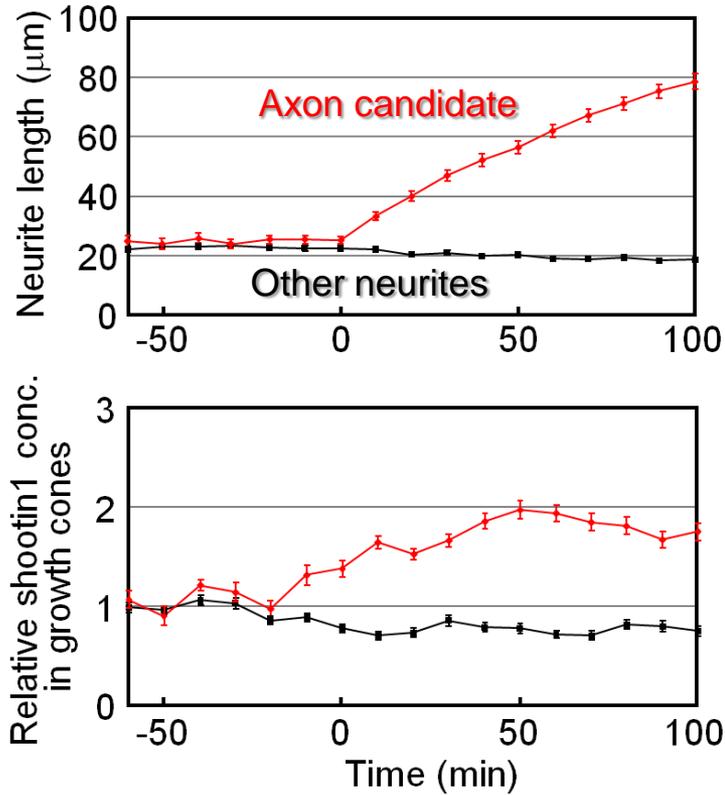
Kaede experiment



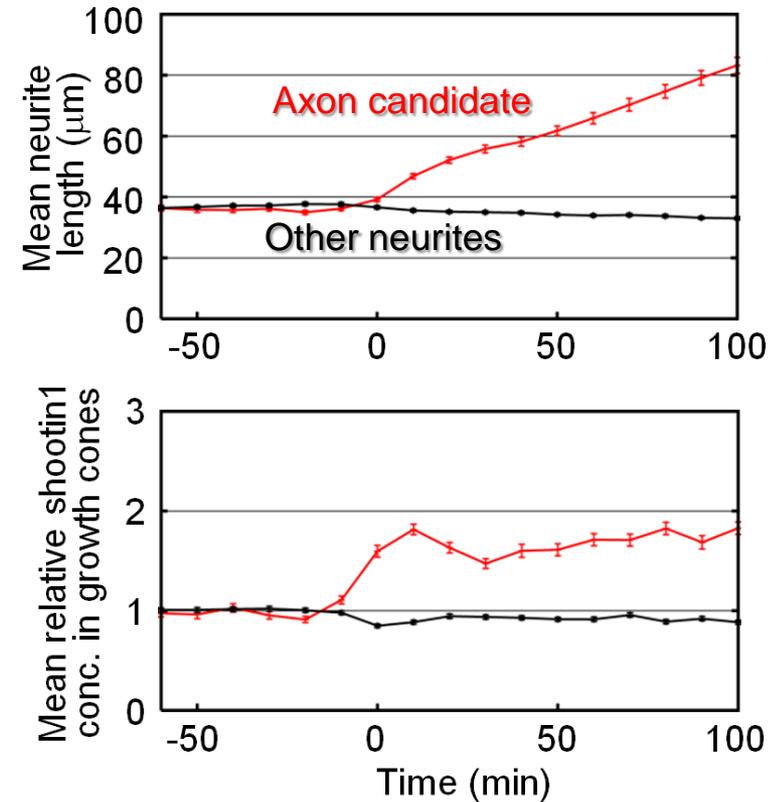
Based on the linear relationship between the neurite length and the decay time constant, we can estimate the diffusion constant.

Experiment vs. simulation

Average time-series(n=9)



Average time-series(n=50)



Experiment: primary cultured neurons

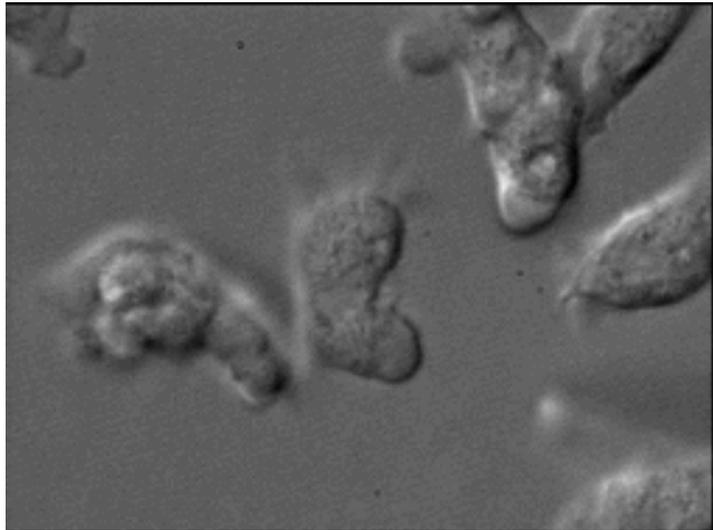
Computer simulation

High SNR in chemotaxis: model and mathematics

Naoki, H., Sakumura, Y., Ishii, S. Journal of Theoretical Biology, 2008.

Spontaneous pulses in chemotactic cells

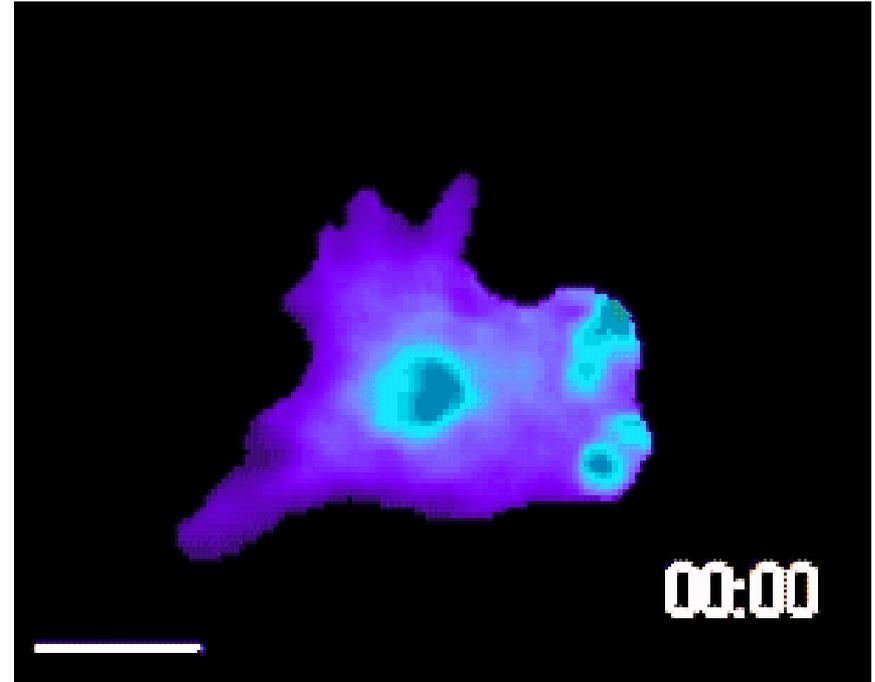
Chemotaxis is a property that a cell detects the gradient of chemical substance and moves up toward its direction.



Cellular slime molds

(from Website of Yanagida Lab in Osaka Univ.)

A chemotactic cell of immune system

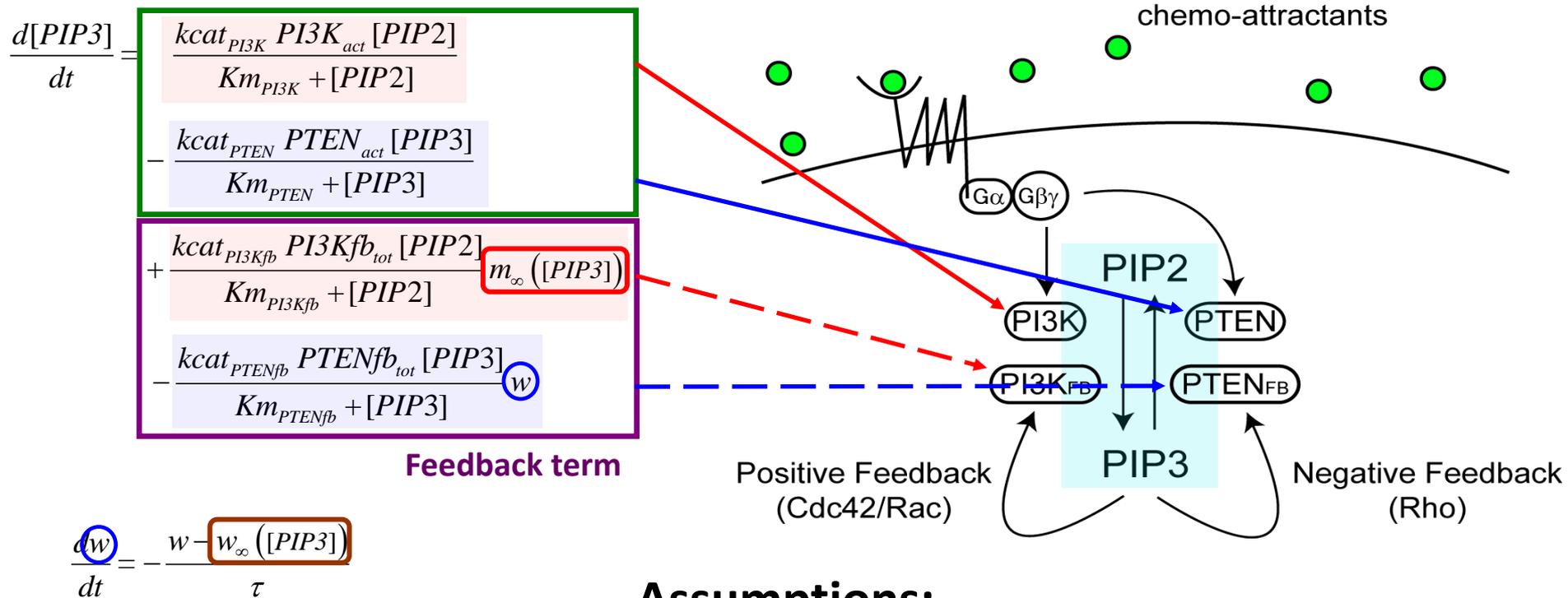


(Arriemerlou et al, Dev Cell, 2005)

PIP3 pulse

Spontaneous and transient increase in *PIP3* concentration leads to cellular elongation and determines moving direction.

A biochemical model of PIP3 pulse generation



Assumptions:

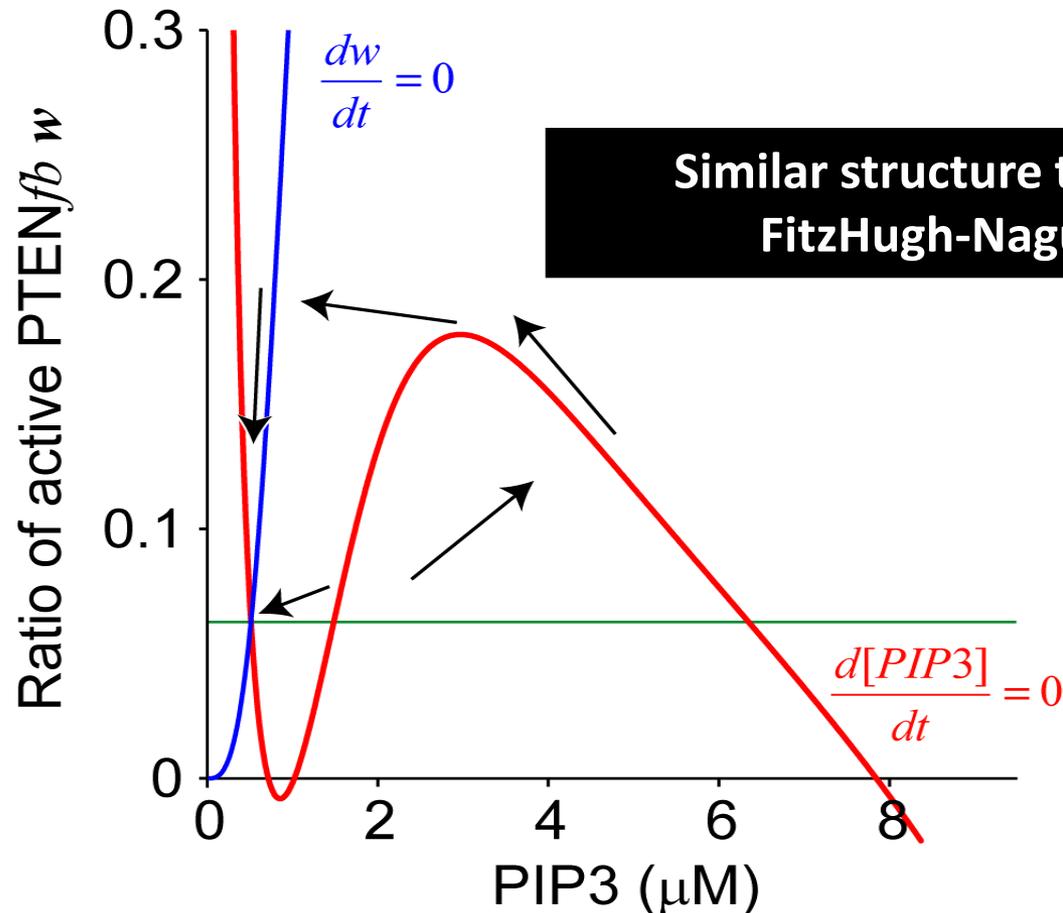
- **Positive feedback is fast and then chemically equilibrated.**
- **Negative feedback is slow.**
- Activities of *PI3Kfb* and *PTENfb* are provided by **Hill equations** of *[PIP3]*.
- Total mass of PIP2 and PIP3 are constant.

where $PIP_{tot} = [PIP2] + [PIP3]$

$$m_{\infty}([PIP3]) = \frac{[PIP3]^{h^{PI3K}}}{K_{PI3Kfb} h^{PI3K} + [PIP3]^{h^{PI3K}}}$$

$$w_{\infty}([PIP3]) = \frac{[PIP3]^{h^{PTEN}}}{K_{PTENfb} h^{PTEN} + [PIP3]^{h^{PTEN}}}$$

PIP3 pathway constitutes an excitable system



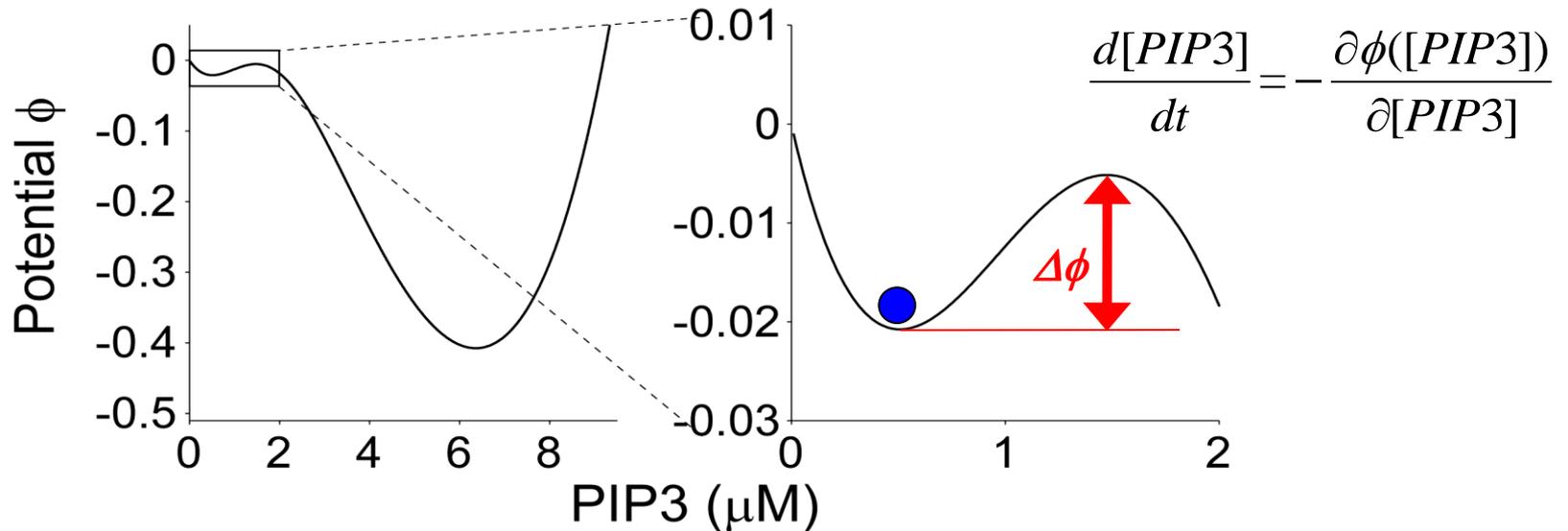
PIP3-related signal transduction consisting of positive and negative feedback loops can be an **excitable system**.

Mechanism of spontaneous signal generation

Brownian motion under potential field

(comprised by stochastic nature of chemical reactions)

Potential:



Langevin equation

$$\frac{d[PIP3]}{dt} = - \frac{d\phi([PIP3])}{d[PIP3]} + \sqrt{2D}\xi$$

$$\langle \xi(t)\xi(t') \rangle = \delta(t-t')$$

Kramers transition rate

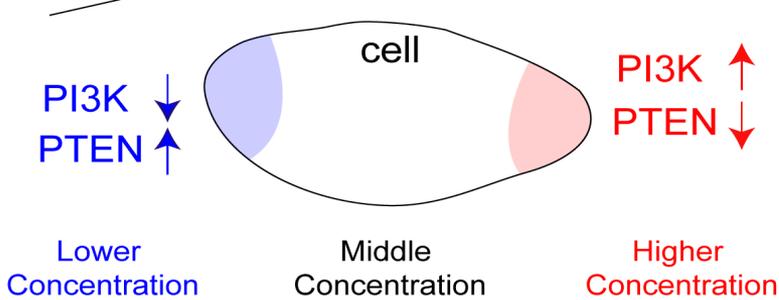
HA Kramers, Physica, 1940

$$k = \frac{\sqrt{|\phi_s'' \cdot \phi_{us}''|}}{2\pi} \exp\left(-\frac{\Delta\phi}{D}\right)$$

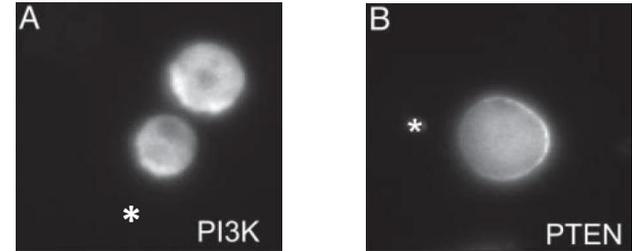
D : Diffusion constant in potential field

Nonlinear amplification of the linear gradient

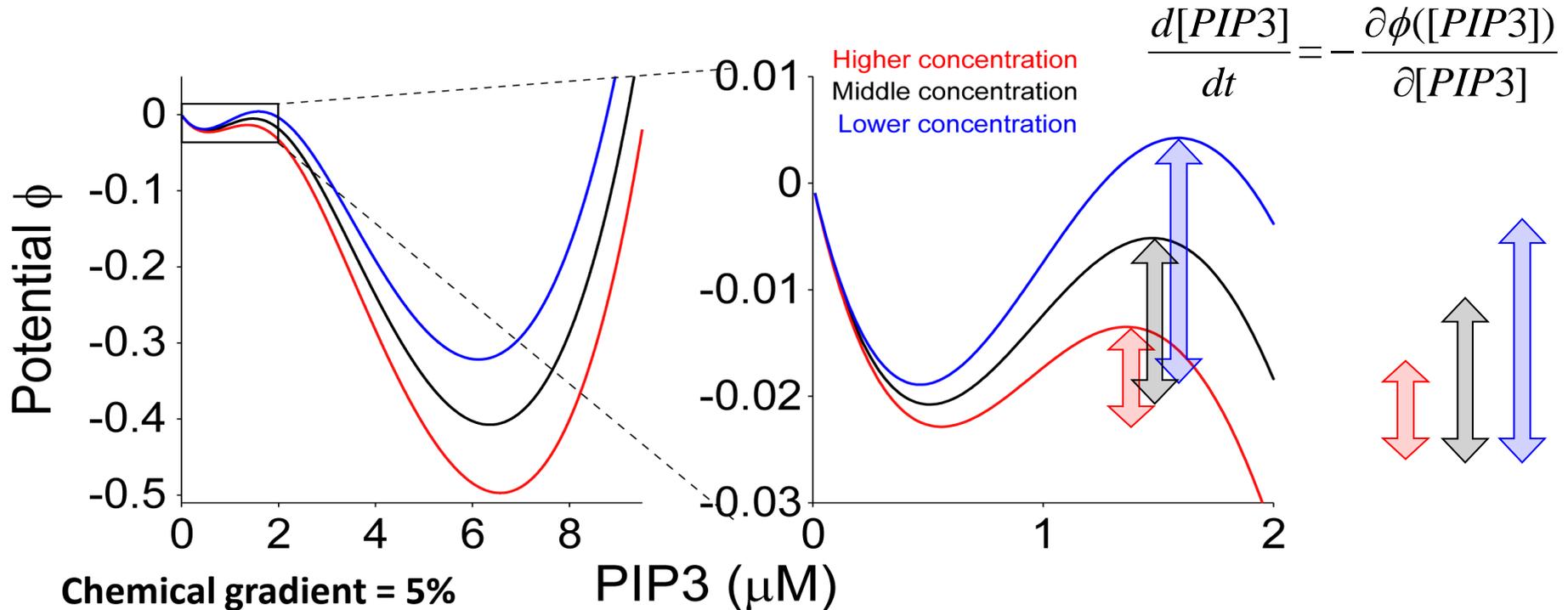
Chemical Gradient



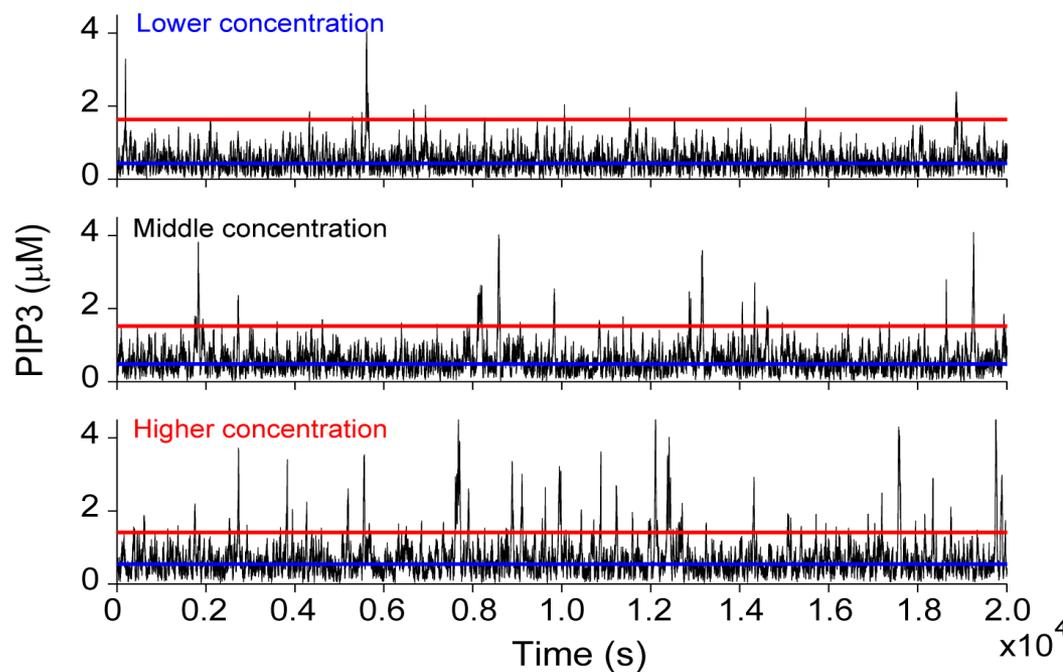
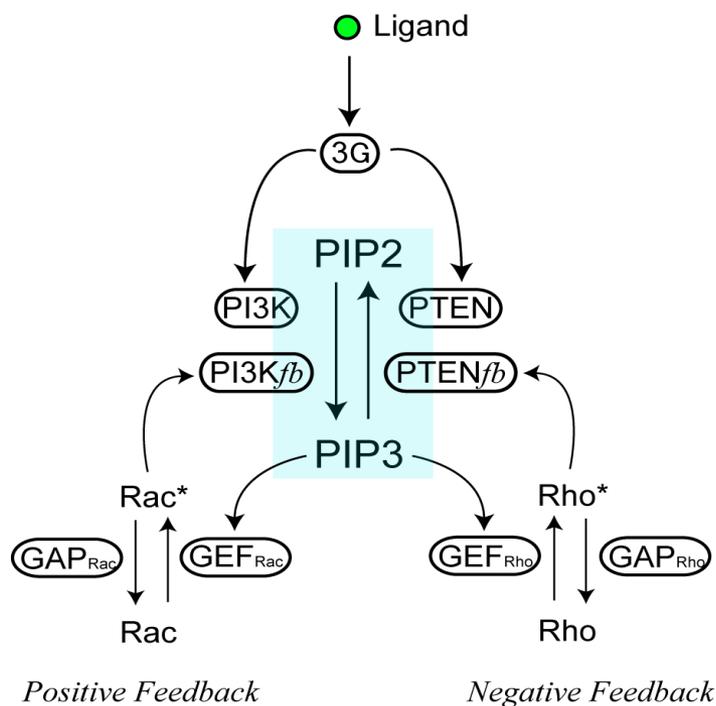
Chemical gradient are give as localized accumulation of PI3K/PTEN



Janetopoulos et al, PNAS, 2004



Monte-Carlo simulation of the signal pathway reproduces the theoretical results



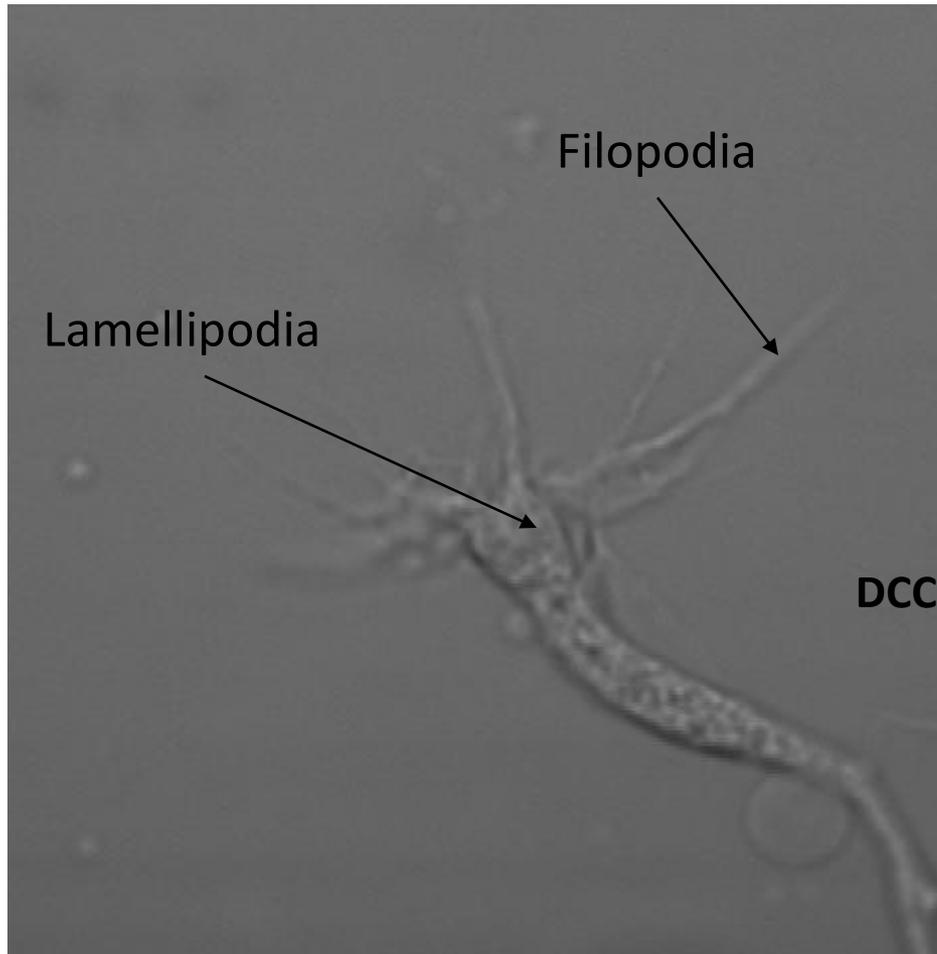
31 molecule kinds / 38 reactions

Monte-Carlo simulation

Bidirectional response by growth cones: biology and model

Nishiyama, M., Naoki, H., Ishii, S., Hong, K.

Bidirectional responses of growth cones

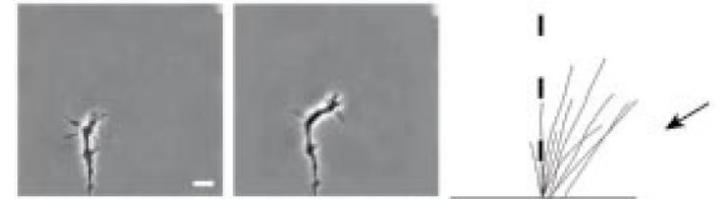


(Xenopus spinal neuron)

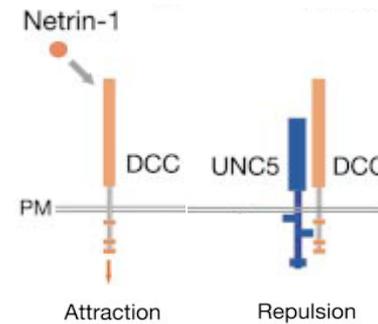
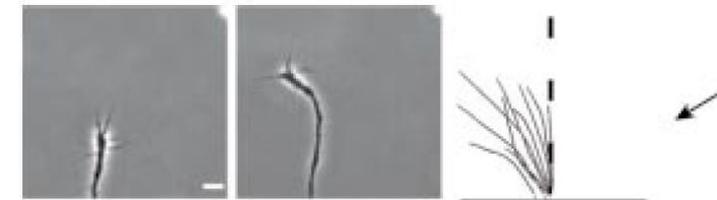
Receptor for cue molecule

Netrin-1 gradient

DCC



DCC-UNC5

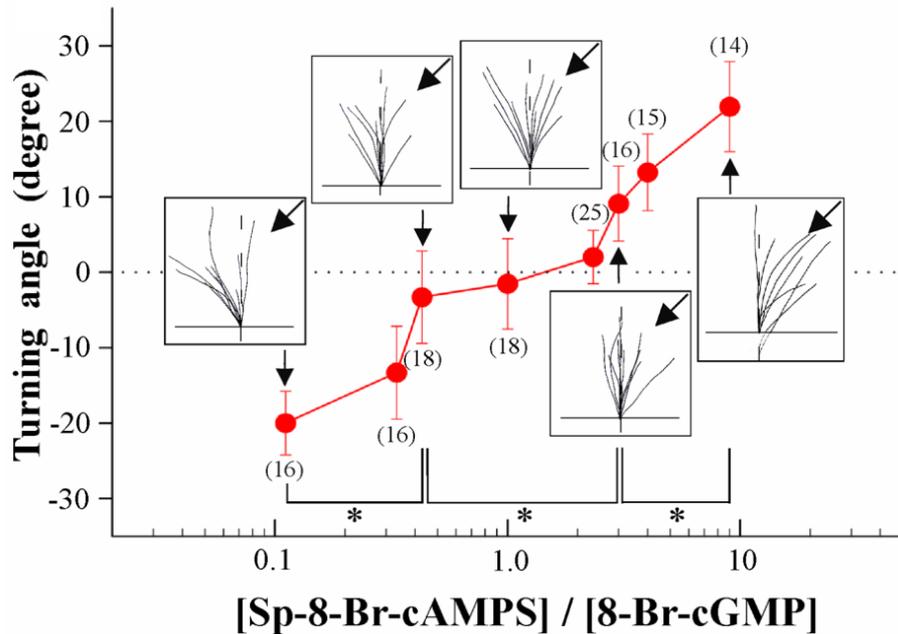
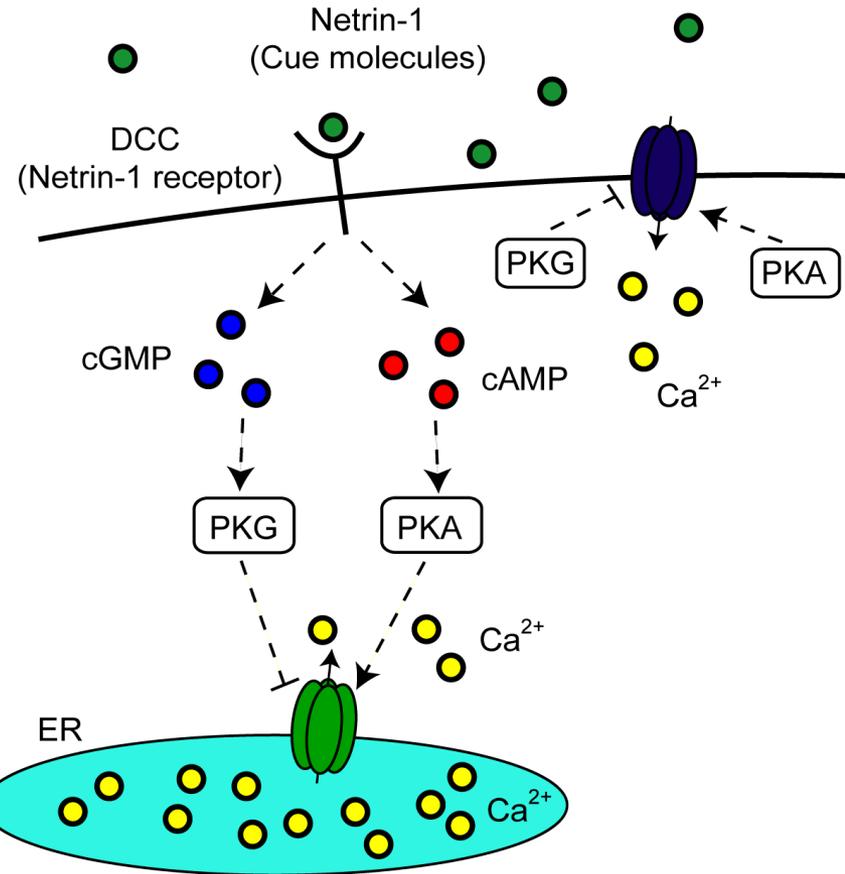
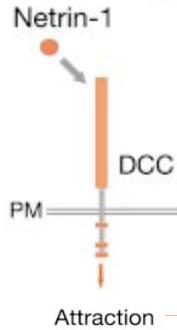


cAMP/cGMP controls bidirectional responses of the growth cone

Netrin-1 gradient

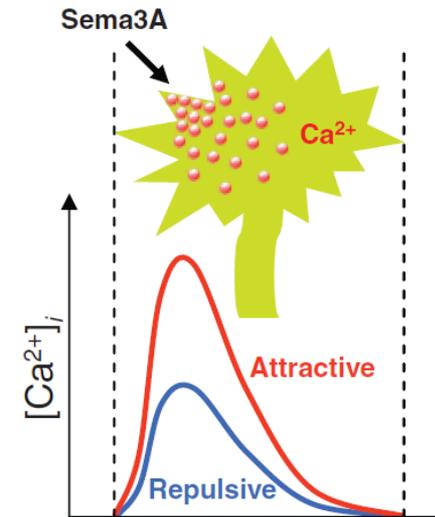
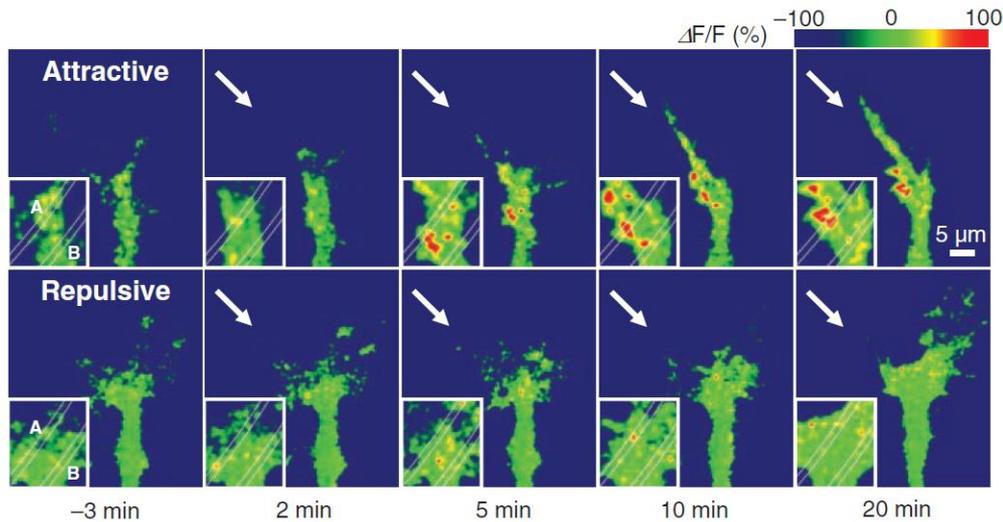


Bath application of Sp-8-Br-cAMPS/8-Br-cGMP



Gradient direction is encoded into steepness or basal concentration of Ca²⁺

Ca²⁺ imaging with Sema3A gradient



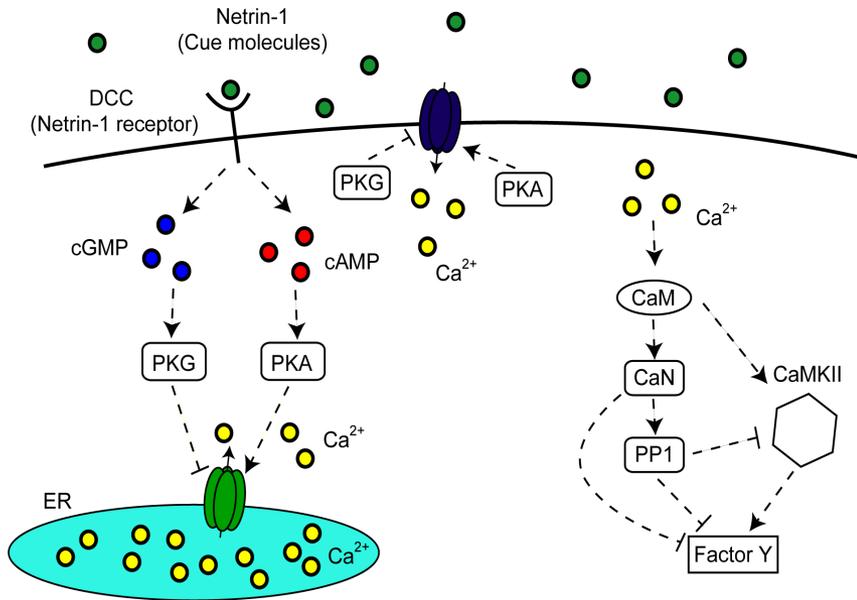
Nishiyama et al., Nat Neurosci (2008)

During attraction, a 2-fold greater Ca²⁺ increase is induced than are induced by repulsive signals.

Questions

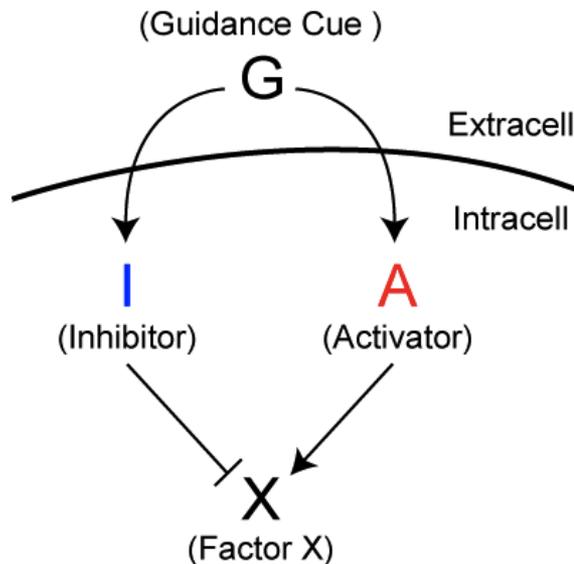
- How gradient information is encoded into Ca²⁺?
- How Ca²⁺ signal is decoded by downstream cascade?

A combined activator-inhibitor model



• In this growth cone turning system, there are two kinds of balancing factors between activator and inhibitor.

- Upstream of Calcium
 - cAMP (A) vs. cGMP (I)
- Downstream of Calcium
 - CaMKII (A) vs. CaN-PP1 (I)

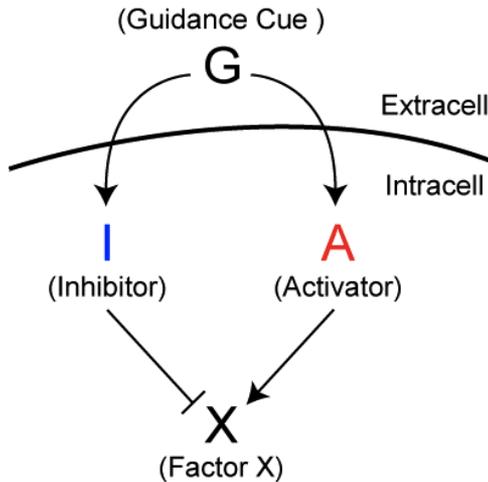


Reaction-diffusion equation

$$\frac{\partial A}{\partial t} = D_A \frac{\partial^2 A}{\partial x^2} + f_A(A; G(x))$$

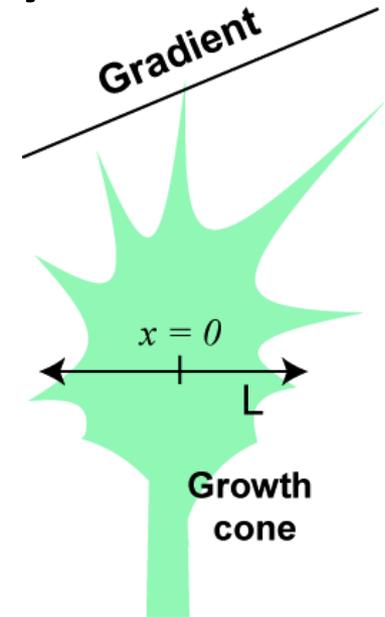
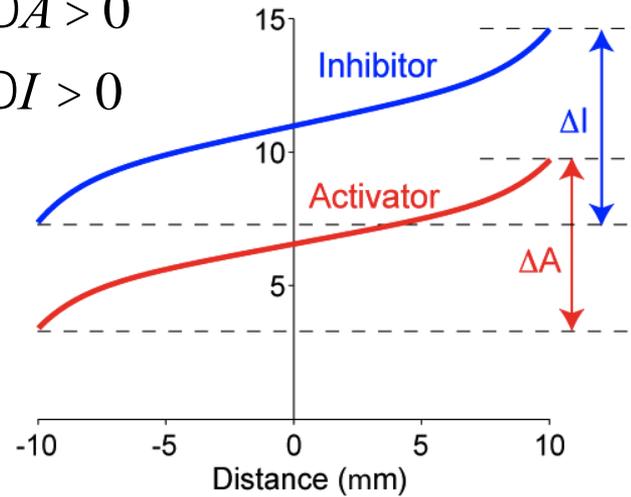
$$\frac{\partial I}{\partial t} = D_I \frac{\partial^2 I}{\partial x^2} + f_I(I; G(x))$$

Mathematics: an activator-inhibitor system can exhibit bidirectionality



$$DA > 0$$

$$DI > 0$$



Spatial difference of X across the growth cone

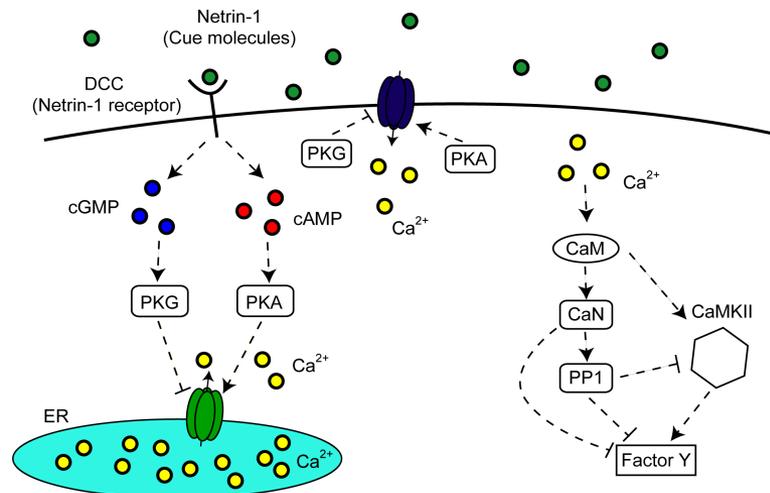
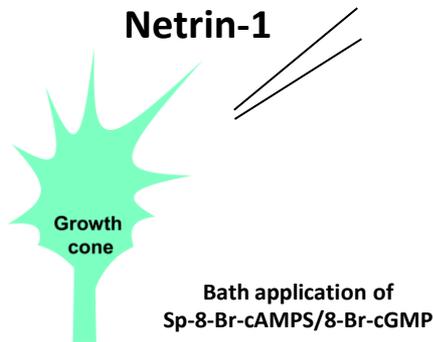
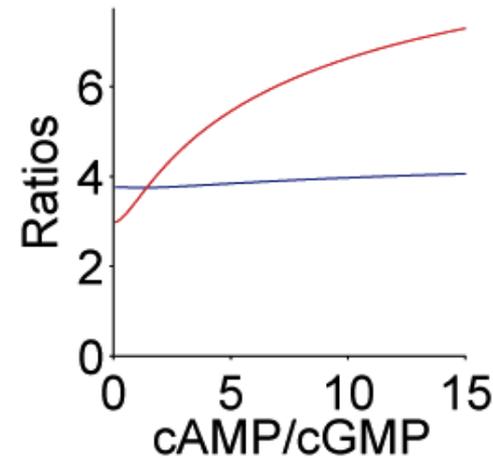
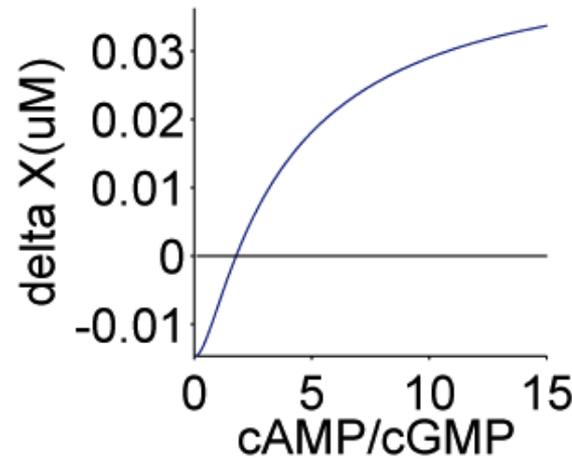
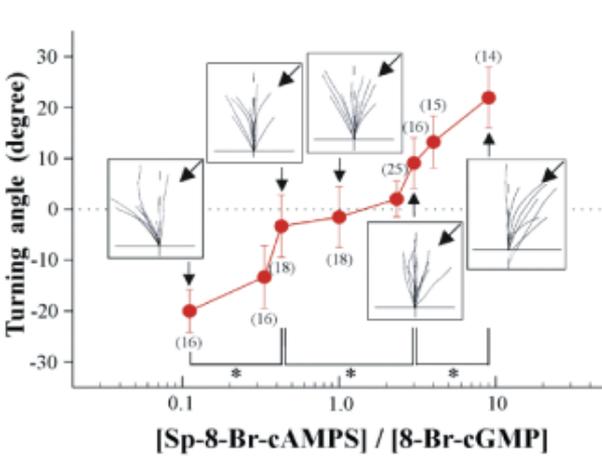
$$\Delta X \equiv X(L/2) - X(-L/2) = \left. \frac{dh_X}{d(A/I)} \right|_{\frac{A^*}{I^*}} \frac{A^*}{I^*} \left(\frac{DA}{A^*} - \frac{DI}{I^*} \right)$$

Repulsive condition ($\Delta X < 0$)

$$\frac{A^*}{I^*} > \frac{DA}{DI}$$

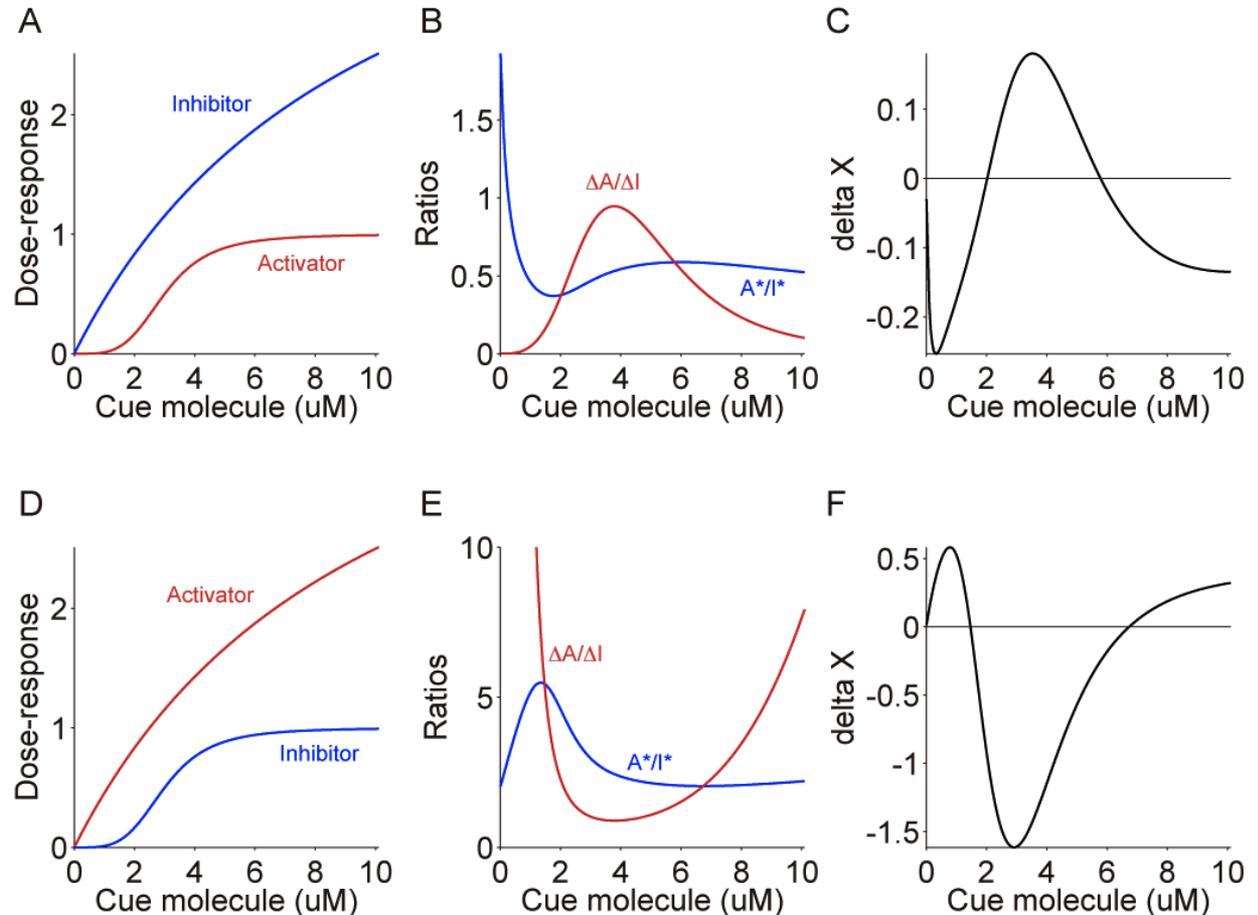
Magnitude relation between A^*/I^* and $\Delta A/\Delta I$ determines attractive or repulsive turnings.

The mathematical model can reproduce bidirectional responses

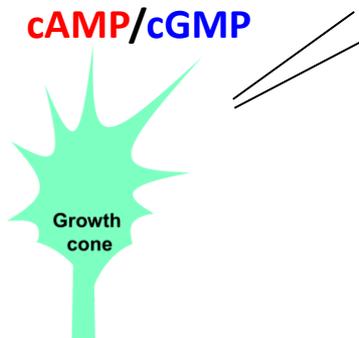
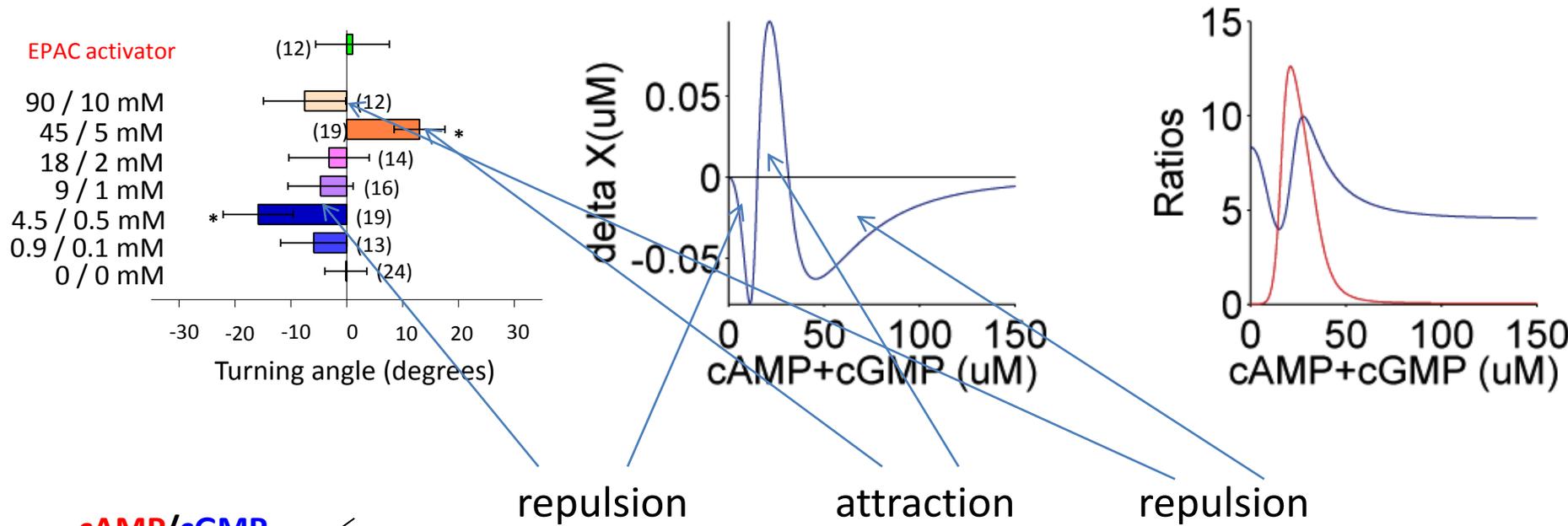


The mathematical model further predicts more complicated behaviors

When the activator and inhibitor are upregulated in a **non-linear** manner,...



The predicted complicated behaviors are experimentally confirmed



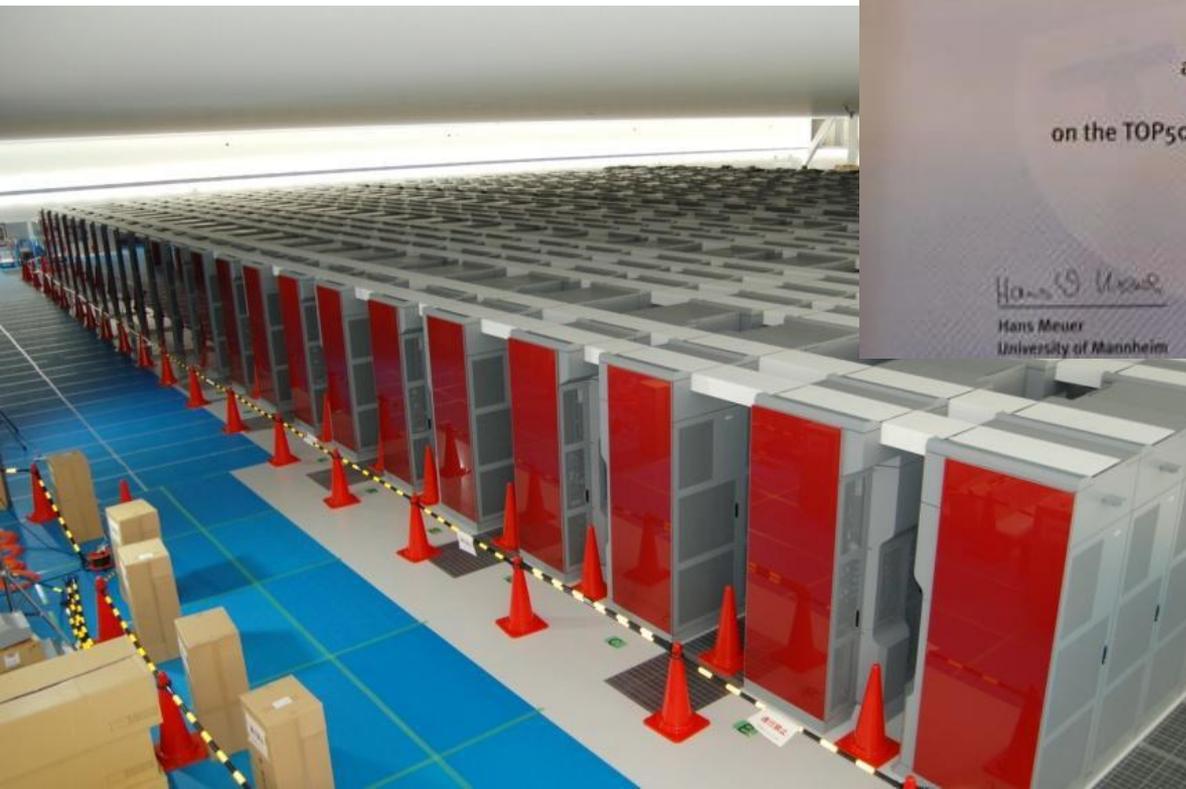
When the activator/inhibitor exhibits non-linear responses, the system's response become complicated. This kind of nonlinearity may be used for situation-dependent growth cone guidance in developing neurons.

Large-scale simulation of neural systems, in Japan

Diesmann, M., Fukai, T., Usui, S., Kuroda, S., Ichikawa, K.,
Kanzaki, R., Doya, K., Ishii, S.,
and many young researchers

Japanese supercomputer K achieved the world fastest (20, June, 2011)

LINPACK performance:
8.162 PFLOPS
(548, 352 cores)



“K” comes from the Japanese word “Kei” which means ten peta or 10 to the 16th power.



K supercomputer outlook

- Over 80,000 processors
 - Over 640K cores
 - Over 1 Peta Bytes memory
- Cutting-edge technologies
 - CPU: SPARC64 VIIIfx, 8 cores, 128GFlops
 - Interconnect, “Tofu”: 6-D mesh/torus
 - Parallel programming environment



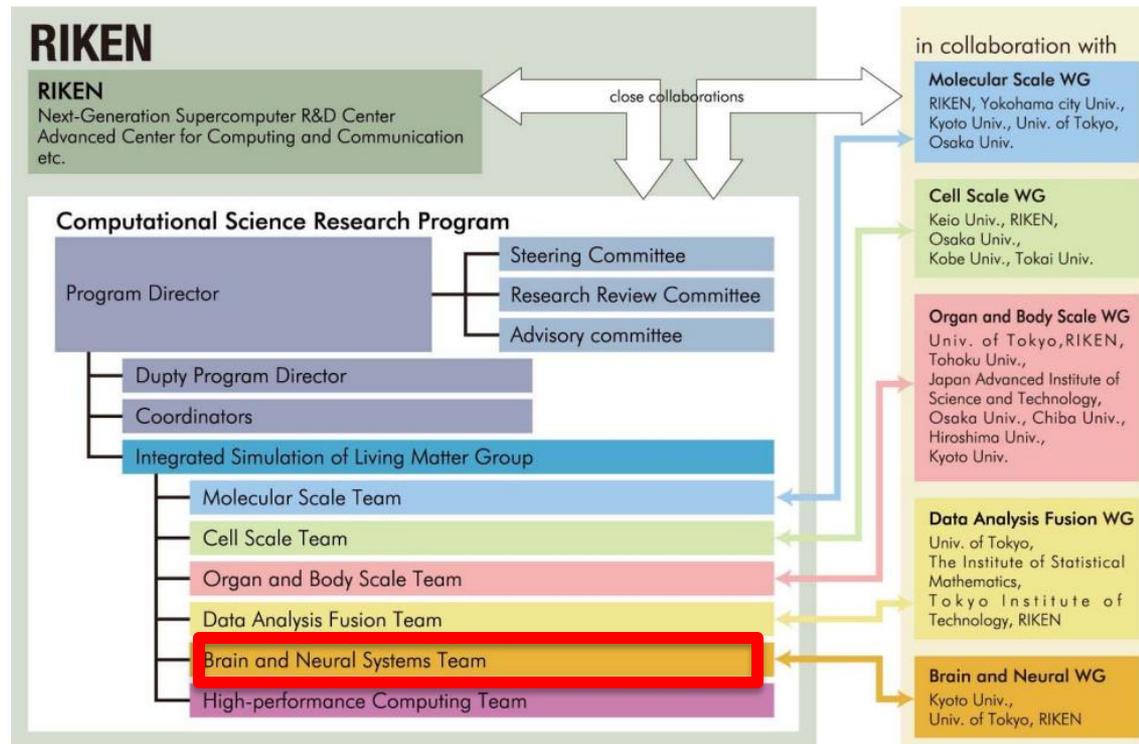
CPU (SPARC64 VIIIfx)	Cores/Node	8 cores (@2GHz)
	Performance	128GFlops
	Architecture	SPARC V9 + HPC extension
	Cache	L1(I/D) Cache : 32KB/32KB L2 Cache : 6MB
	Power	58W (typ. 30 C)
	Mem. bandwidth	64GB/s.
Node	Configuration	1 CPU / Node
	Memory capacity	16GB (2GB/core)
System board(SB)	No. of nodes	4 nodes /SB
Rack	No. of SB	24 SBs/rack
System	Nodes/system	> 80,000

Inter-connect	Topology	6D Mesh/Torus
	Performance	5GB/s. for each link
	No. of link	10 links/ node
	Additional feature	H/W barrier, reduction
	Architecture	Routing chip structure (no outside switch box)
Cooling	CPU, ICC*	Direct water cooling
	Other parts	Air cooling

<http://www.fujitsu.com/downloads/TC/sc10/when-high-performance-computing-meets-energy-efficiency.pdf>

Grand challenge: Next generation integrated simulation of living matter

- Promote the research and development of simulation software which helps **understand phenomena from molecules to entire organisms**
- Long-term "grand challenges" aimed at the construction of a basis for future life science unifying experiments and computer simulations to gain new knowledge for the first time.



http://www.csrp.riken.jp/index_e.html

Software: neuron and circuits simulators

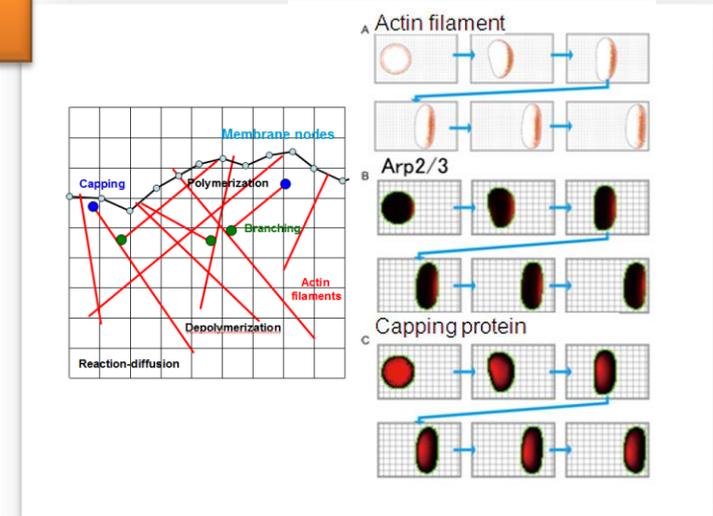
NeuroMorphoKit

C, C++, MPI, OpenMP, GSL, NetCDF, GD, zlib

A multiphysics simulation environment for neuromorphological dynamics

is a software platform for neuronal morphological simulation by integration of kinetics of cytoskeletal filaments, cell membrane dynamics, and reaction-diffusion of intracellular molecules

Shin Ishii
Kazuhisa Ichikawa



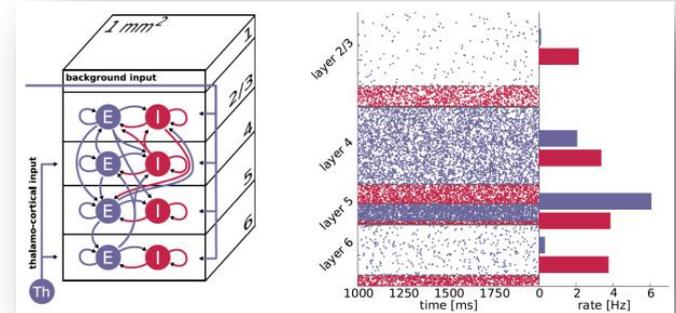
NEST

C++, SLI, MPI, pthread

Neural Simulation Tool

simulates and predicts the signal processing for 10 million neurons equivalent to 100 columns in the cortex, and 100 billion synapses connecting the neurons

Markus Diesmann
Tomoki Fukai



Neural Simulation Tool (NEST) on K

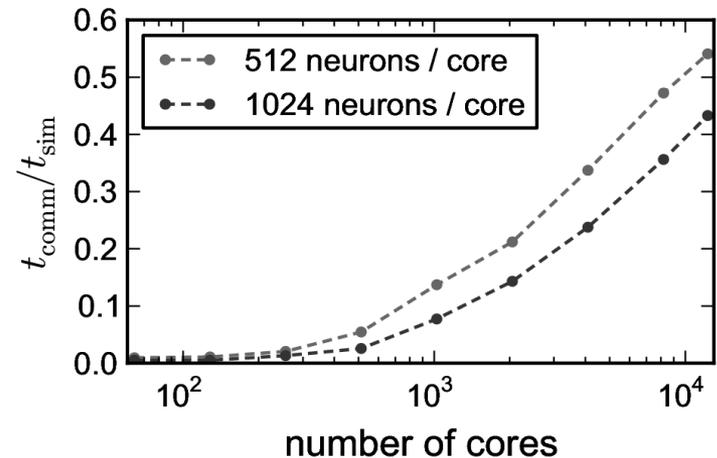
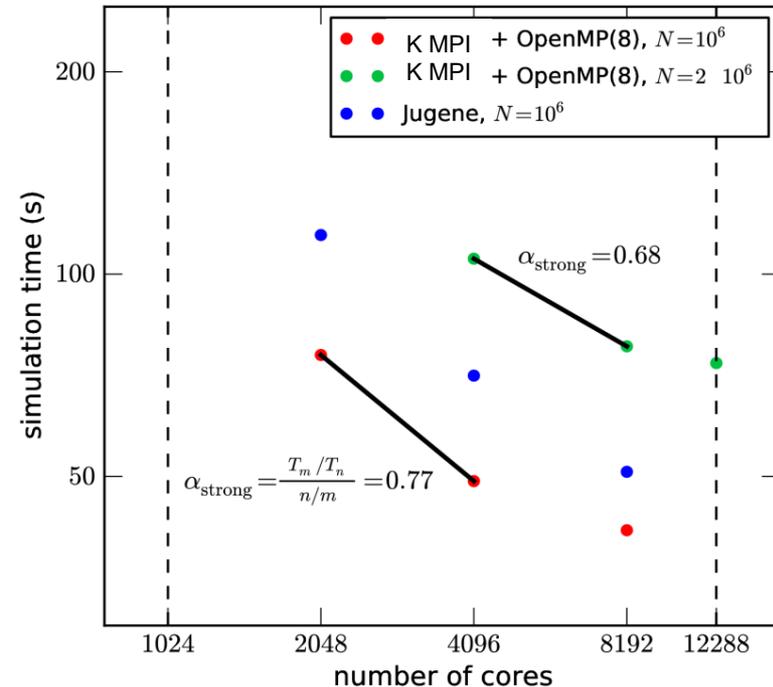
Now available on K

- high degree of parallelization is achieved by using hybrid MPI + OpenMP threads with more than 8000 cores on K supercomputer
- very good scaling up to 4096 cores, speedup $\alpha > 0.75$
- good scaling for > 8000 cores, speedup $\alpha = 0.68$

Action plan

- Employ computer's specific optimizations of NEST code
- improves communication computation balance

NEST STDP benchmark, $K = 10^4$



Software: whole-brain level circuits

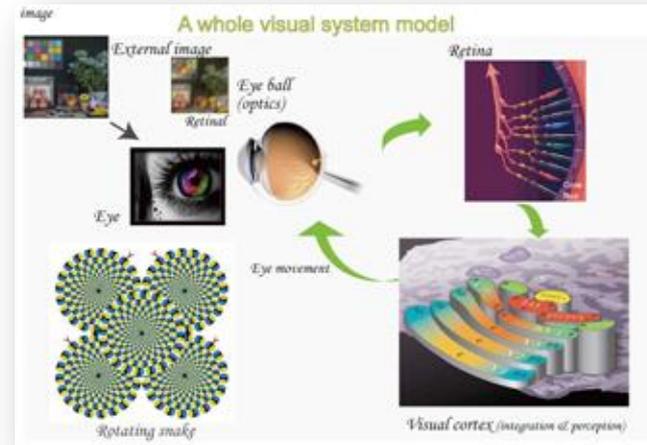
VSM

C, C++, OpenMPI, GSL, netCDF

The visual information processing analysis with a whole visual system model

targets the visual system being built with the mathematical model that is described in each level of function, cell, and ion current for cortex, retina, ophthalmological optics, and eye motion (brainstem)

Shiro Ushi, Kenji Doya
Shinya Kuroda



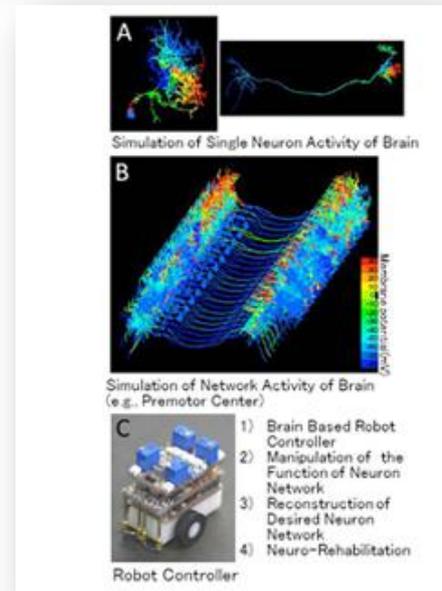
IOSSIM

C, C++, MPI, SUNDIAL InterView

Whole-brain simulator for the insect's olfactory system

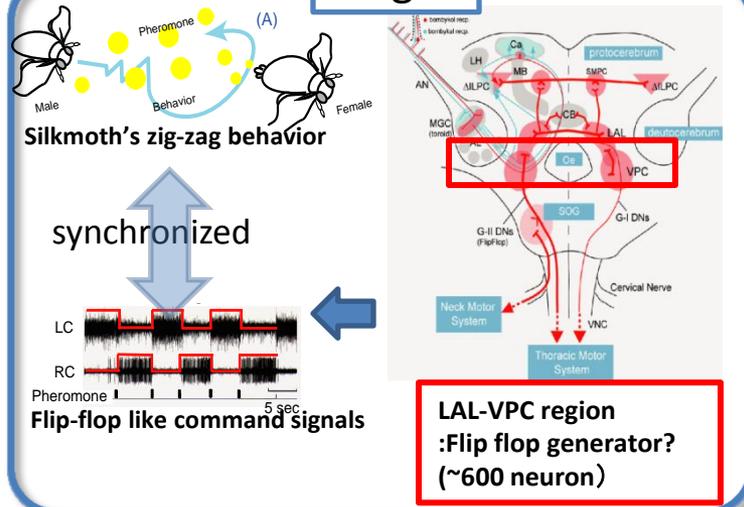
performs a virtual-spatial real-time simulation for the neural circuit's information processing of an insect from sensing to action by the multi-compartment model that considers each neural configuration

Ryohei Kanzaki

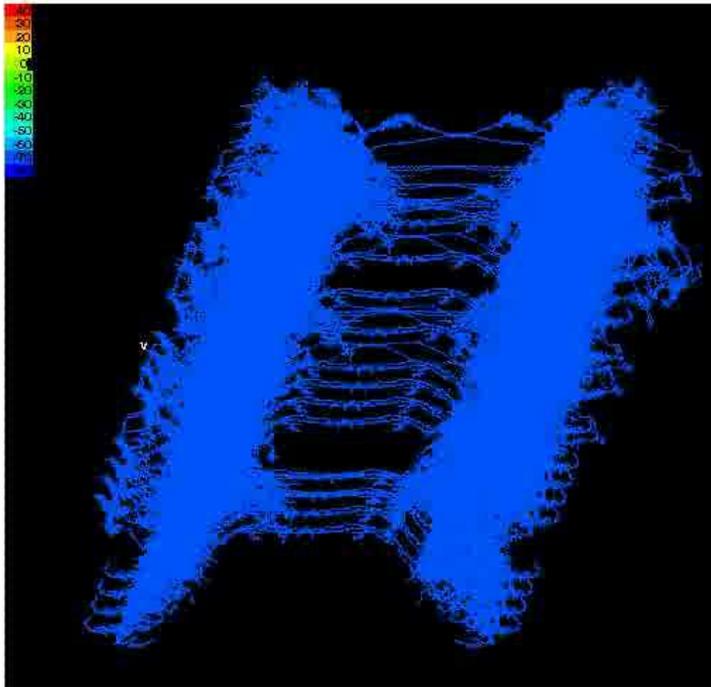
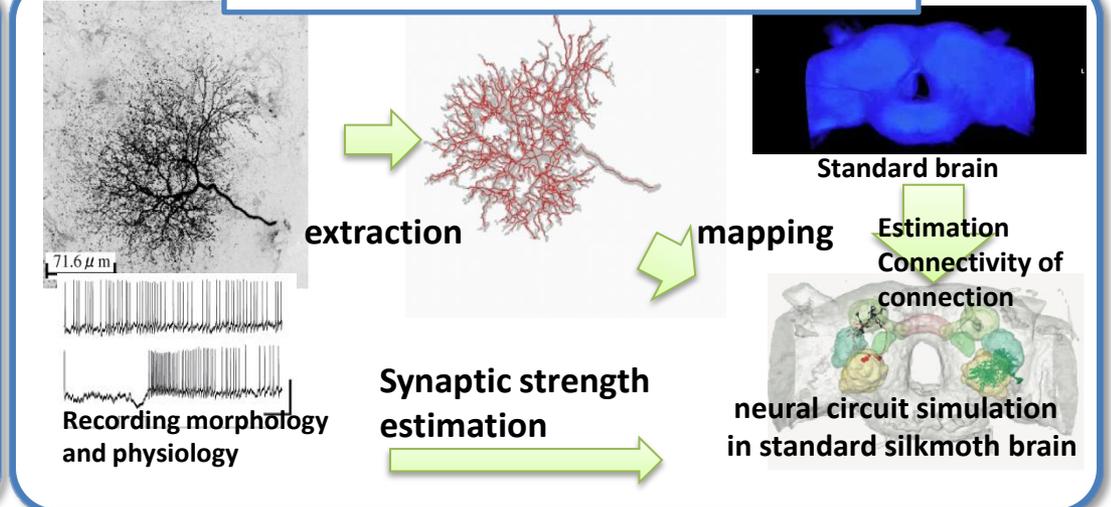


Large-scale simulation of insects' whole olfactory system (IOSSIM)

Target

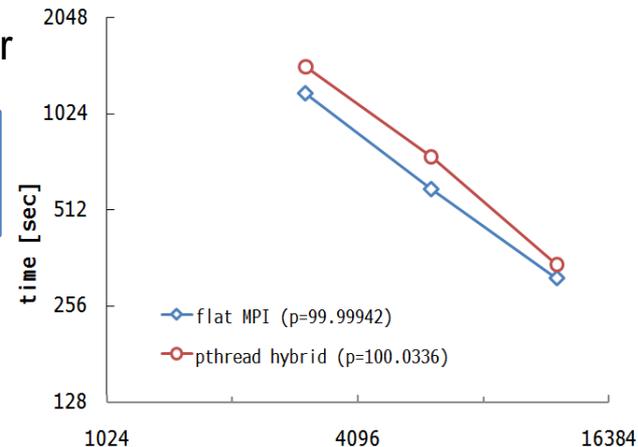


Methods to construct simulation



- Multi-compartment H-H neurons
- 72 neurons, 12900 synapses
- Inhibitory linter-neurons and excitatory bilateral neurons
- Synaptic connections based on morphological analysis of the LAL-VPC circuit
- Working on K computer

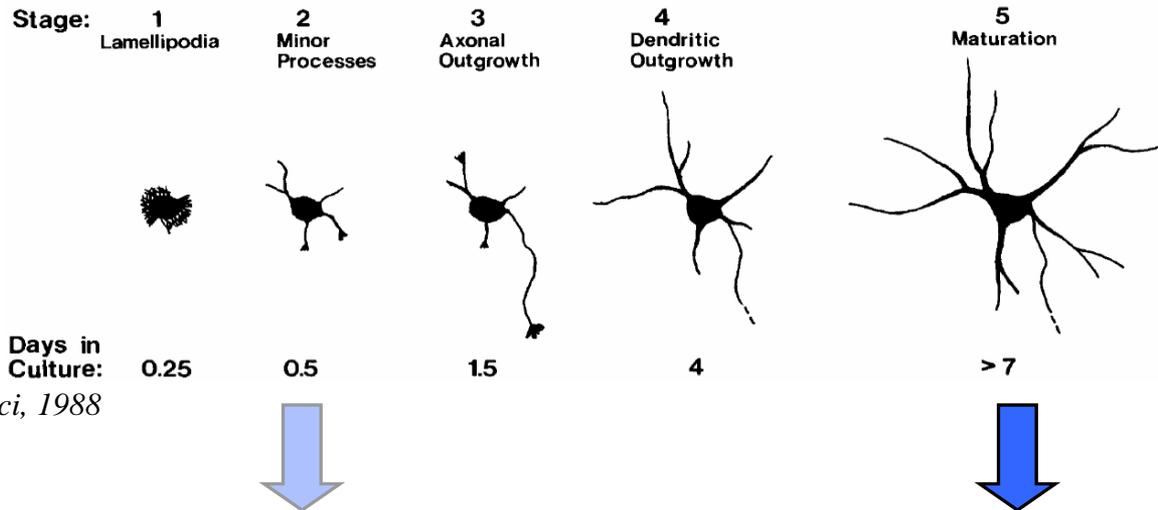
Large-scale simulation on K supercomputer



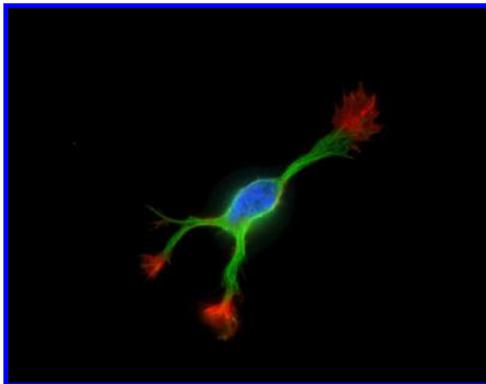
Cytoskeleton-based morphogenesis: a multi-physics simulation

Nonaka, S., Naoki, H., Ishii, S. Neural Networks, 2011.

Cytoskeleton in neuronal morphogenesis

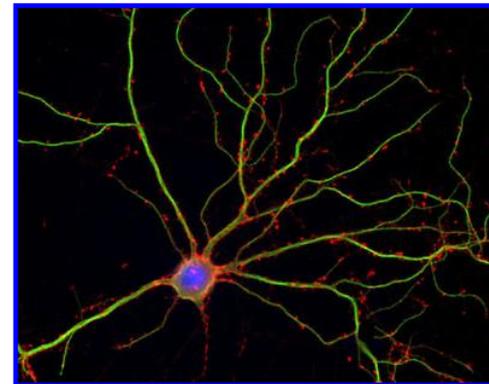


Actin filament
Microtubule



Neurites:

Actin and **Microtubule** are localized at tips and along shafts, respectively.



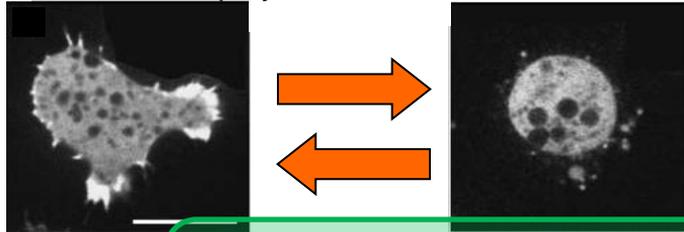
Dendrite / Synapses:

Actin and **Microtubule** are localized at synapses and along dendritic shafts, respectively.

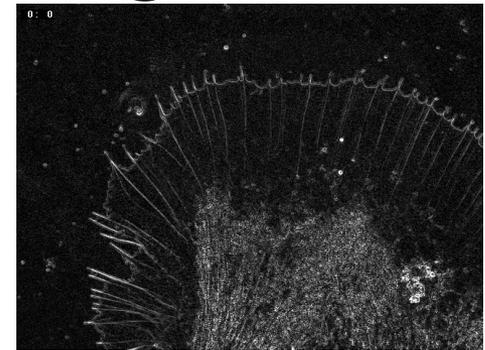
Cytoskeleton, especially actin, is involved in structural plasticity.

Multi-physics in cellular morphogenesis

Actin polymerization inhibitor



Gerisch, et al., Biophys J, 2004



(by Dr. Kaoru Katoh)

Membrane
Deformation and motility

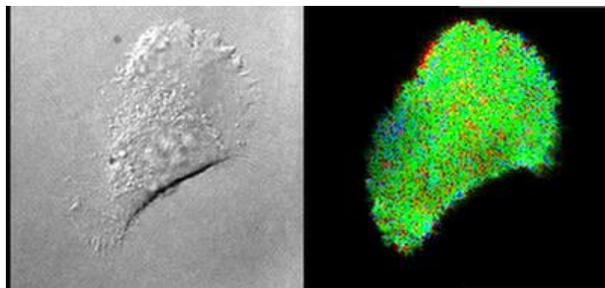
Mechanically sustain membrane

Boundary conditions for
reaction-diffusion

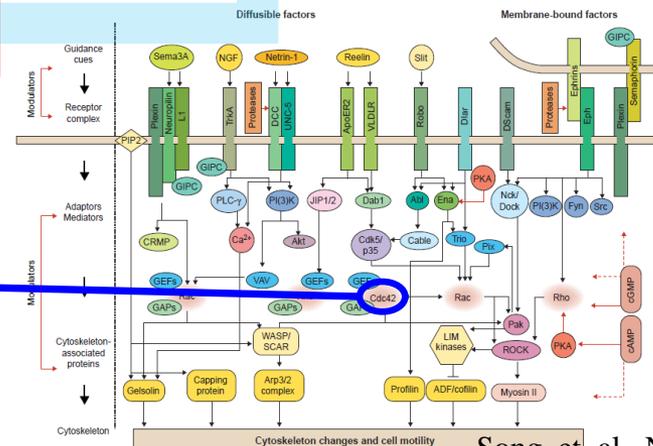
Cytoskeleton (actin)
Polymerization, blanching, etc

Intracellular signaling
Reaction-diffusion

Kinetics control



Cdc42-FRET imaging in fibroblast by Matsuda lab

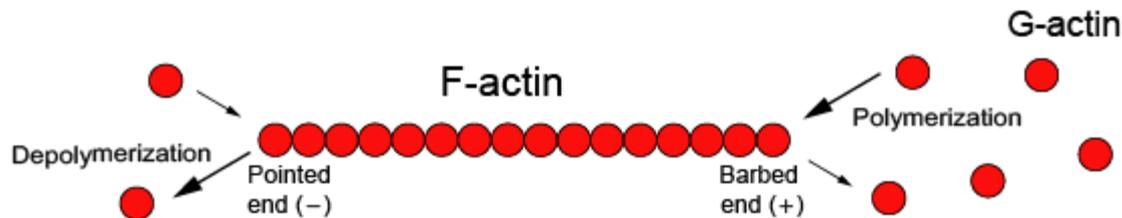


Song, et. al., Nat Cell Biol, 2001

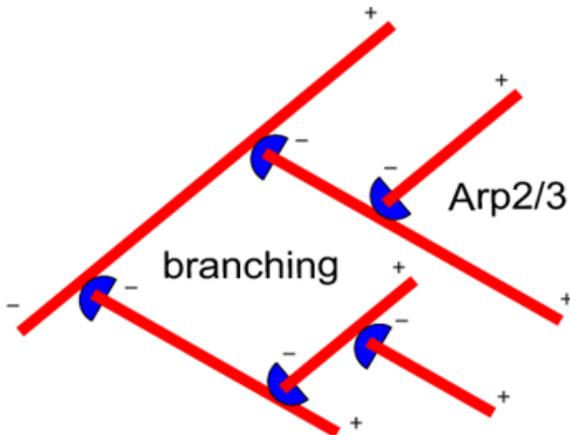
Actin filaments control cell motility

Polymerization and de-polymerization
Treadmilling

Actin treadmill would provide
driving force for cell motility



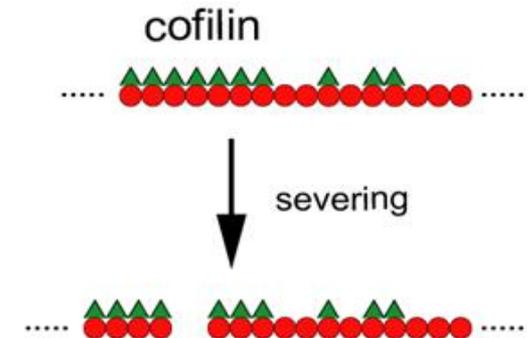
Branching



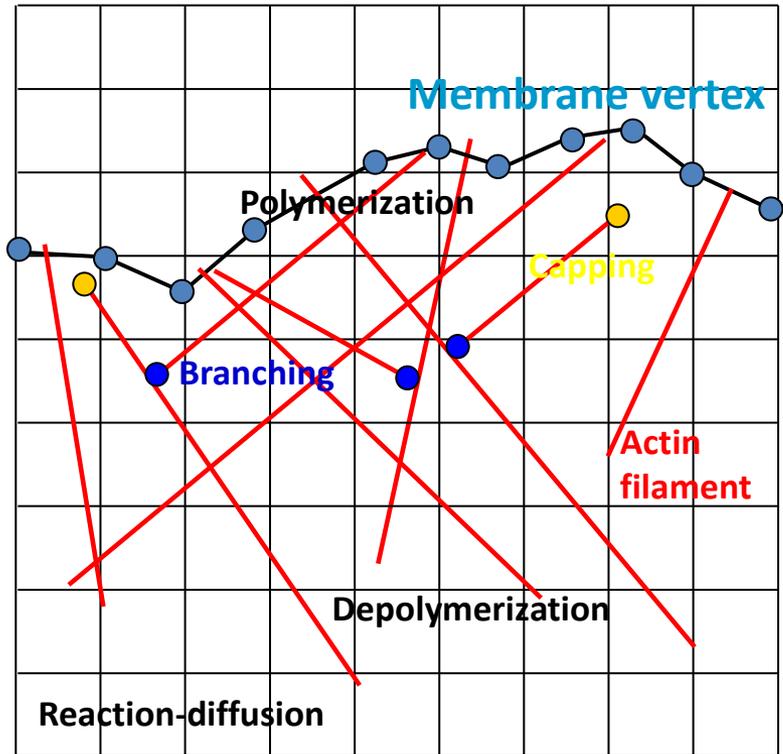
Capping



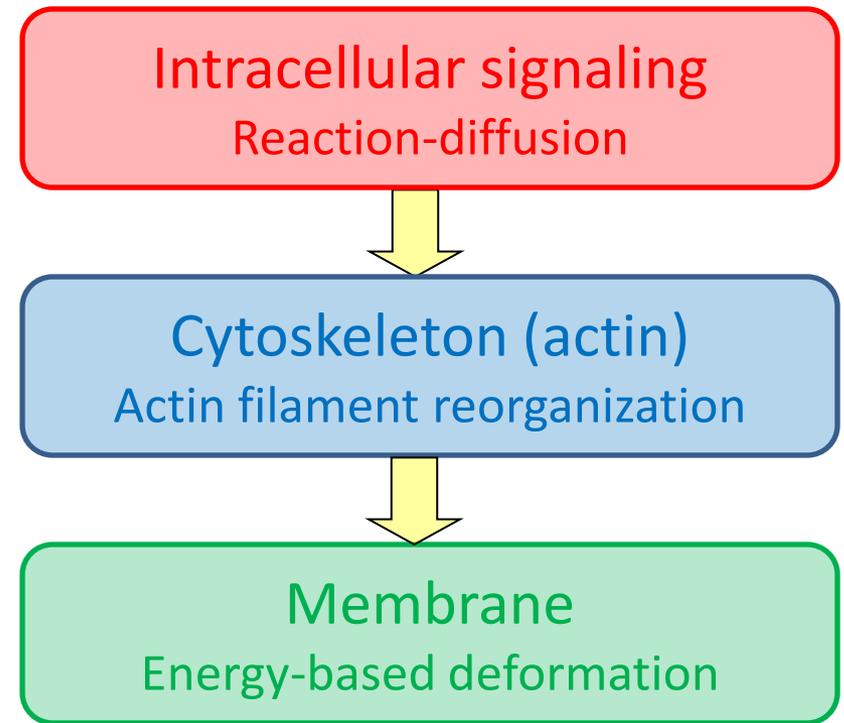
Severing



Compartment model of multi-physics



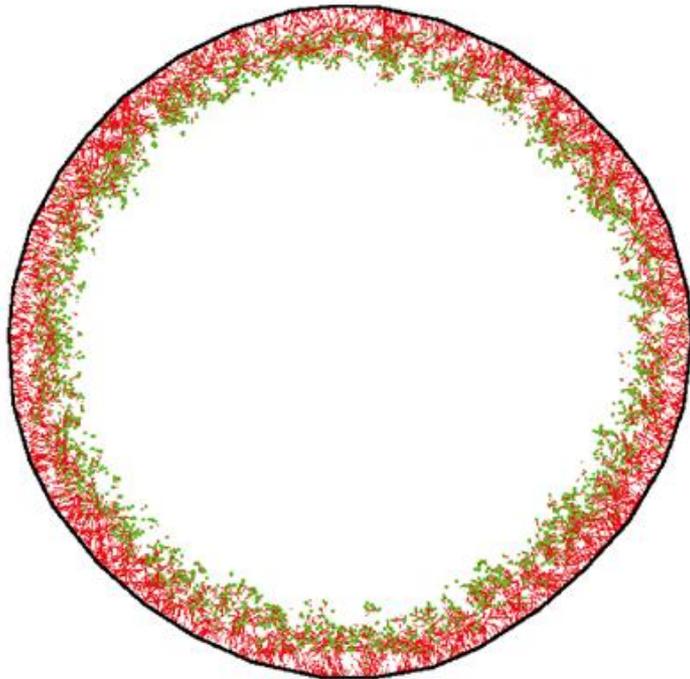
Within each step (Δt)



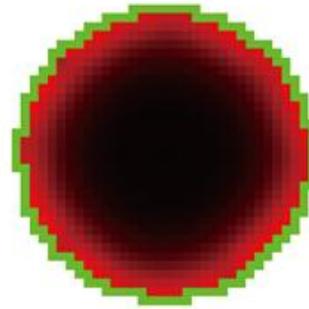
- Space is compartmentalized.
- Membrane is expressed by polygon.
- Actin filament is expressed by line segments.

Simulation: self-organization of lamellipodia

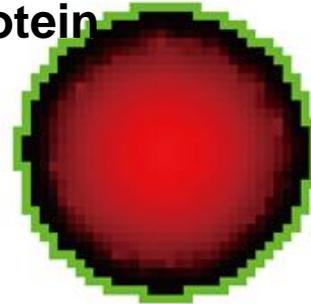
Actin filament
Capping protein



Active Arp2/3



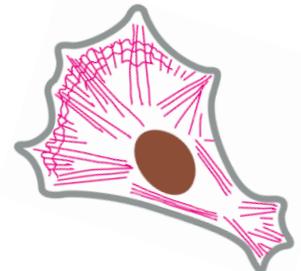
Active Capping protein



- Arp2/3 is activated in the vicinity of the membrane.
Blanchoin, et. al, Biophys J, 2005

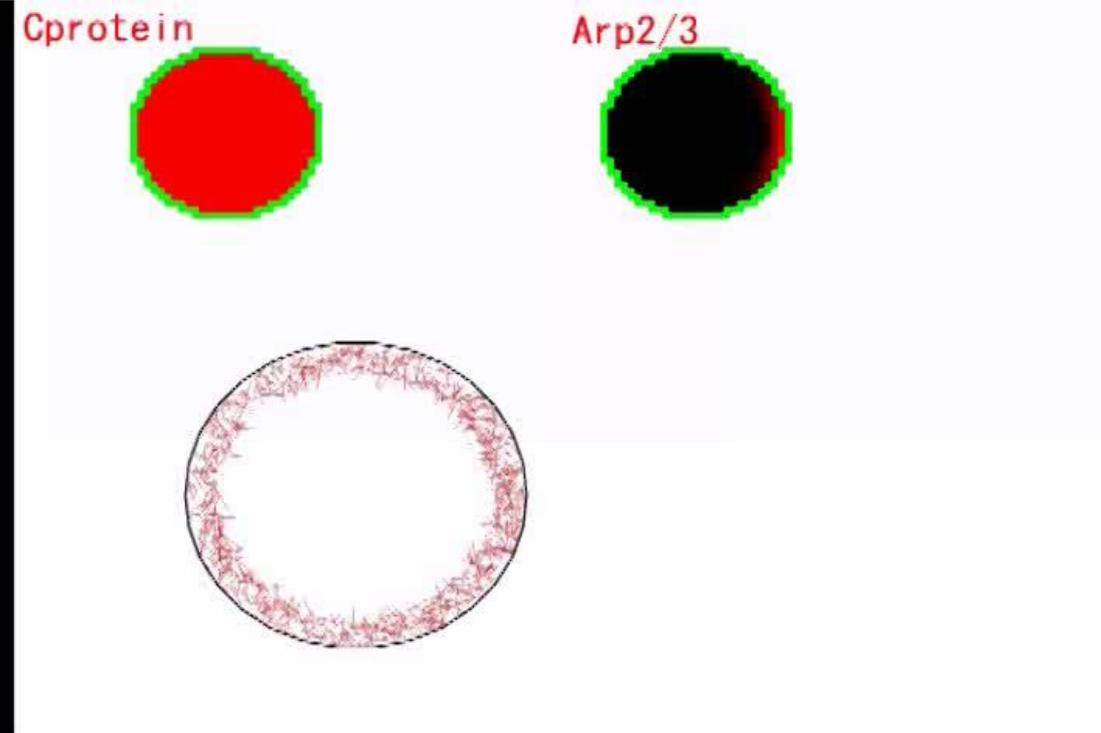
- Capping protein is inactivated in the vicinity of the membrane.
Bear, et. al, Cell, 2002
Schafer, et. al, J Cell Biol, 1996

Meshed network of actin filament is organized.



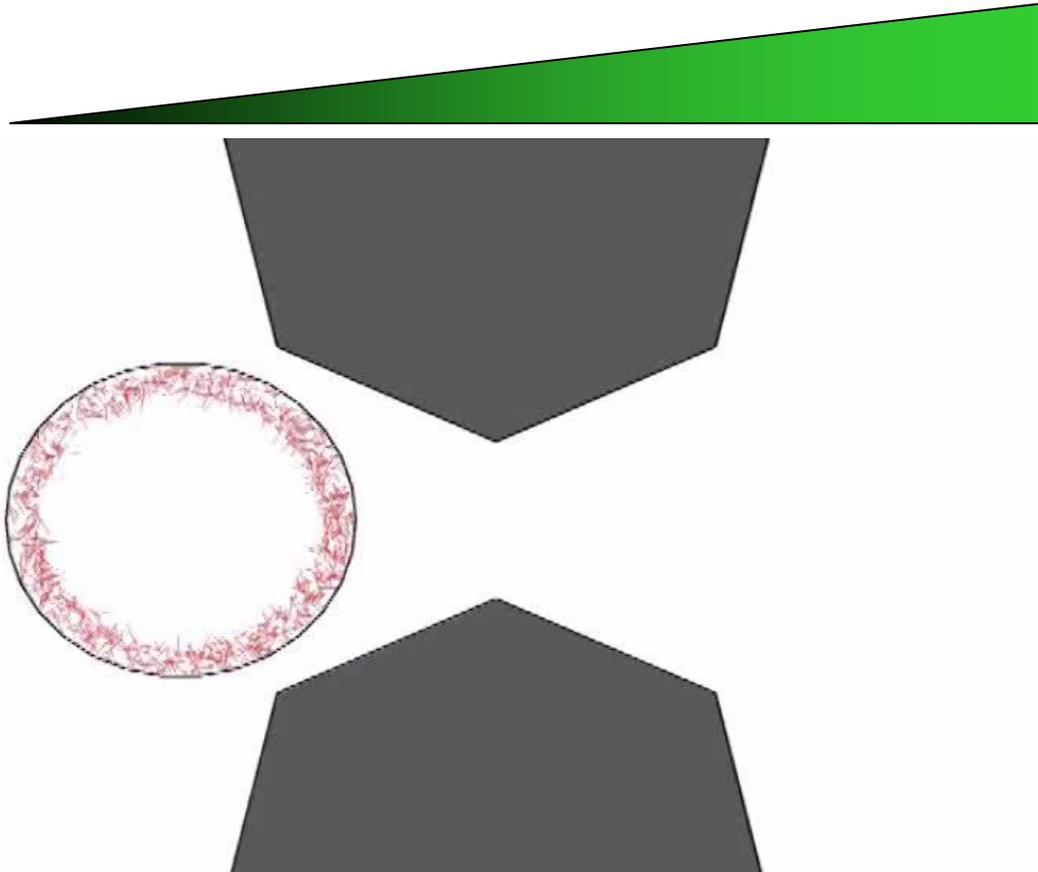
Simulation: chemotactic migration

Chemo-attractant gradient



- Chemo-attractant activates receptor.
- Activated receptor activates Arp2/3 complex.

Simulation: invasive migration



Chemo-attractant gradient

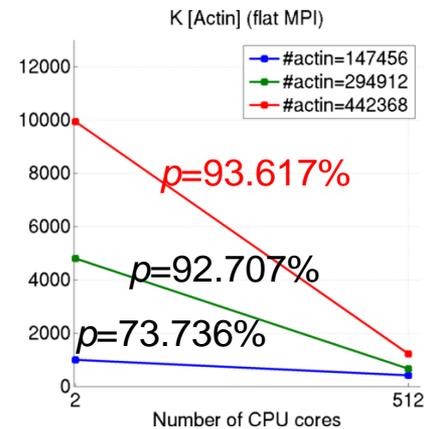
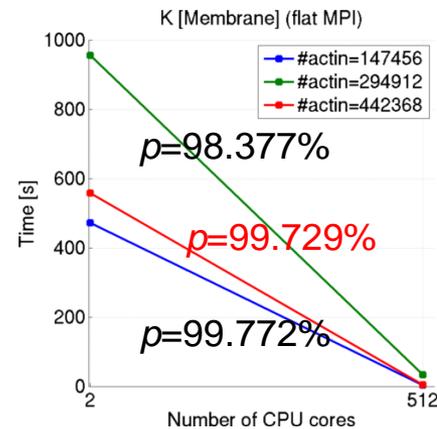
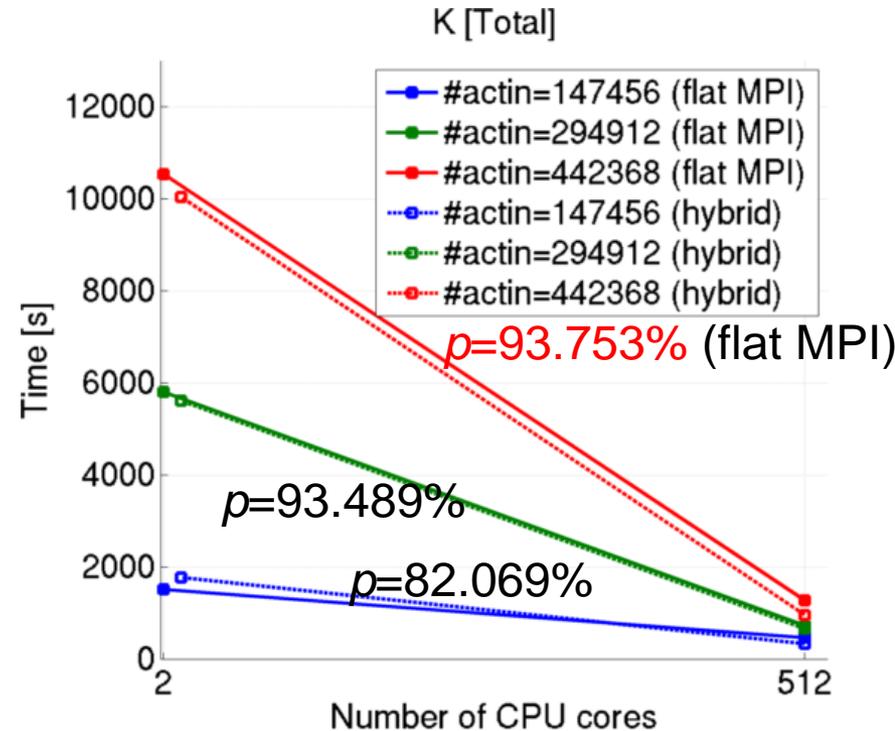
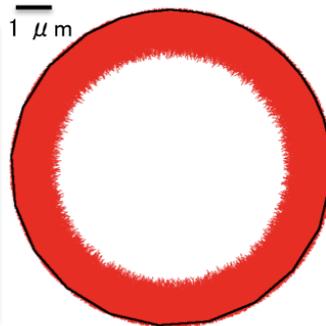
- Locate obstacles, which correspond to other cells or extra-cellular matrix.
- Energy optimization is performed with a constraint such that the membrane vertices are not overlapped with the obstacles.

Simulation on K supercomputer

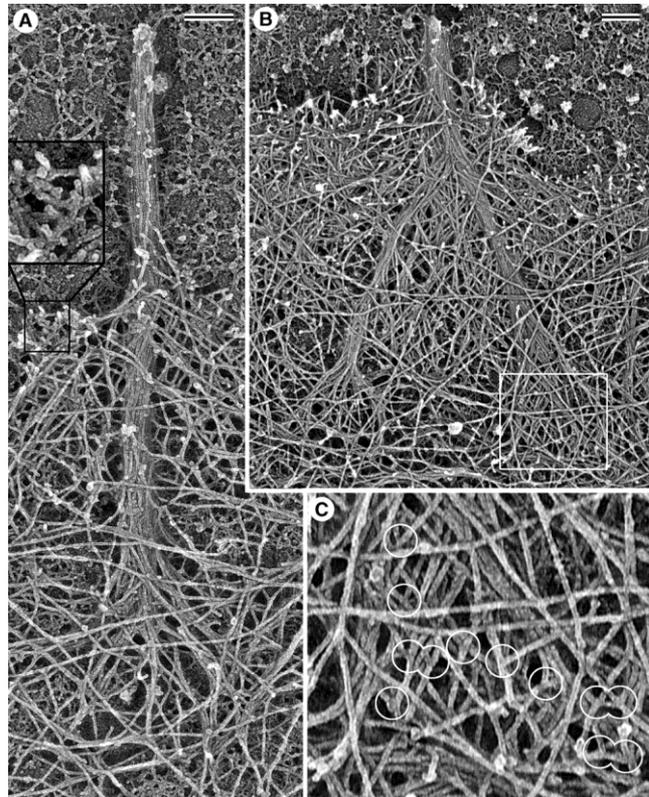
- Current status
 - tested up to 512 cores
 - hybrid of MPI and OpenMP enabled more efficient computations on larger scale settings
 - accomplished moderate parallelization on the membrane energy optimization ($p=99.729\%$)

Simulation setup

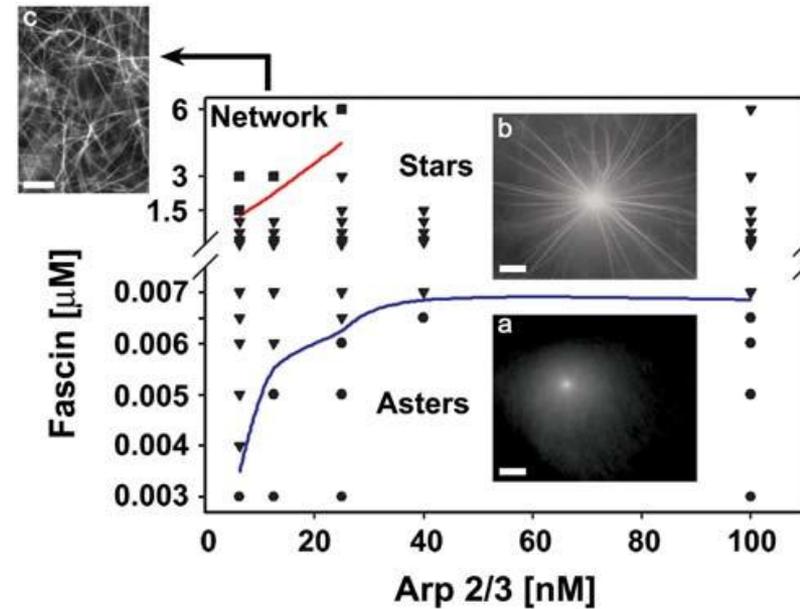
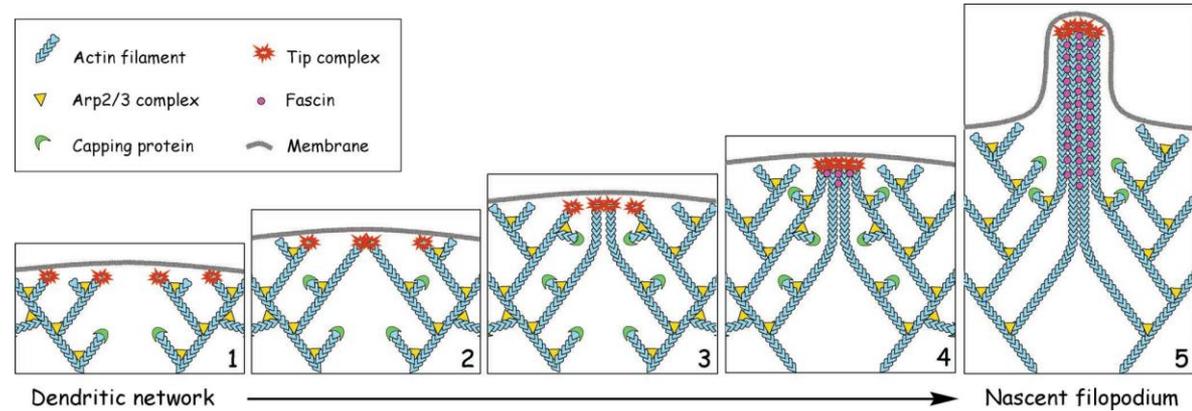
- initial number of actin filaments $> 10^6$
 - small: 147456
 - middle: 294912
 - large: 442368
- initial number of membrane nodes
- 1200 ms biological time simulation



Bendable F-actin and linker protein are required for filopodial formation



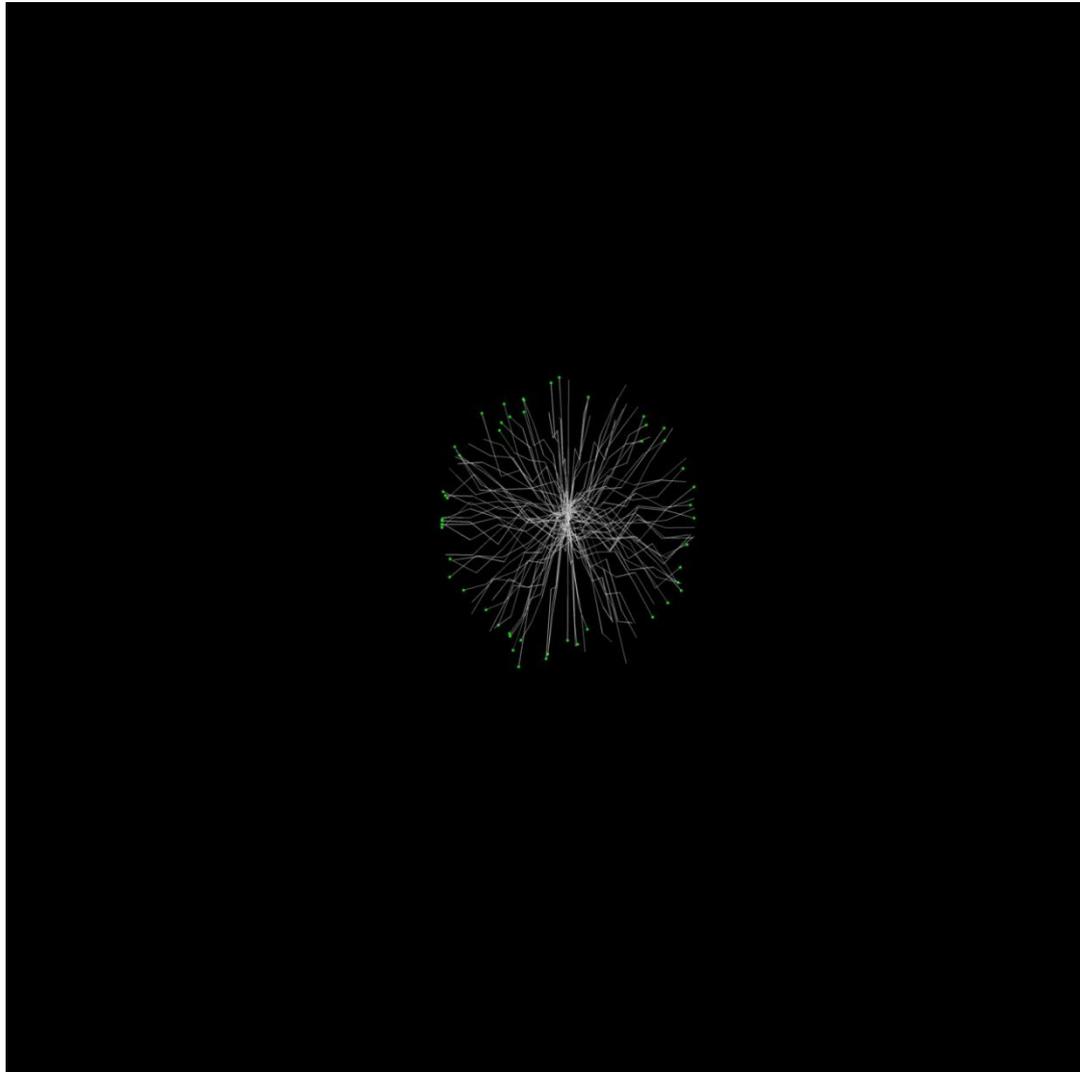
Svitkina *et al.*, J Cell Biol, 2003



Ideses *et al.*, PLoS ONE, 2008

Filopodia-like structure appears in a membrane-free environment only with actin, Arp2/3 and fascin.

Simulation of *in vitro* reconstruction of filopodia



Star-like structure of filopodia is self-organized

While line: bendable F-actin Red point: fascin

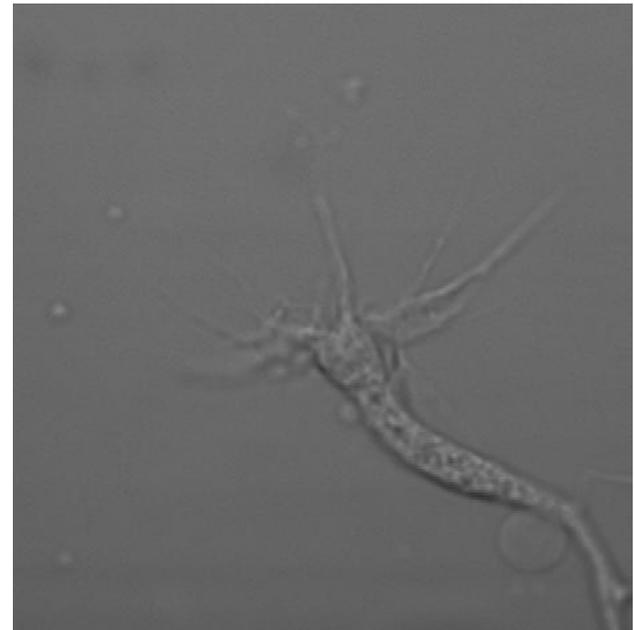
Future: from modeling to decoding

In house



Brain Machine Interface
(decoding from the brain)

In vivo



Growth-cone Machine Interface?
(decoding from neurons and growth cones)

Summary

- Information processing in neuronal morphogenesis
 - Neuronal polarization
 - High SNR in chemotaxis
 - Bidirectional responses by growth cones
- Large-scale simulations of neural systems
 - Large-scale simulation studies in Japan
 - Multi-physics simulation of neuronal morphogenesis
 - Lamellipodia formation and cell migration
 - Bendable F-actin and filopodia formation