FALL 2004

MATH 221   **FINAL EXAM**   **PASSMAN**

NAME _____

T. A.'s NAME _____

**Do all 8 problems and show all work.
Otherwise full credit will not be given.**

PROBLEM	GRADE
1 20 pts	
2 20 pts	
3 20 pts	
4 20 pts	
5 20 pts	
6 20 pts	
7 20 pts	
8 20 pts	
TOTAL	

1. a) [10 points] Compute
$$\lim_{x \to 0} \frac{1 + x^2 - \cos x}{x \sin x}$$

b) [10 points] Find the Riemann sum for the function $f(x) = 3x$ if the interval $[0, 2]$ is partitioned into n subintervals of equal length and if the sample points are the midpoints of each small interval. You can use the fact that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

2. a) [10 points] Find the derivative with respect to x of

$$\int_1^{x^3} \sqrt{1 + t^2} \, dt$$

b) [10 points] Evaluate

$$\int_0^{\sqrt{\pi/2}} x \cos(x^2) \, dx$$

3. a) [10 points] Find the area of the "triangular" region bounded on the left by the y-axis, above by the curve $y = \sqrt{3}\cos x$ and below by $y = \sin x$.

b) [10 points] The region bounded below by the parabola $y = x^2$ and above by the line $y = 2x$ is rotated about the x-axis. Find the volume of this solid of revolution.

4. a) [10 points] Differentiate the following function and simplify your answer.

$$\ln\left(x + \sqrt{x^2 + 5}\right)$$

b) [10 points] Evaluate the integral below. Write your answer as $\ln r$ for some real number r.

$$\int_0^2 \frac{2x^3}{x^4 + 9}\, dx$$

· 5. a) [10 points] Find all values of x where the derivative of the following function is equal to 0.

$$x^4 \cdot \exp(x^2 - 6x + 1)$$

b) [10 points] Compute the integral below and write your answer as $\ln r$ for some real number r.

$$\int_0^{\ln 3} \frac{e^x}{e^x + 1} \, dx$$

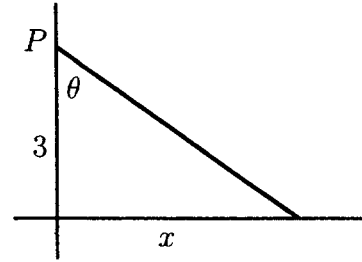6. a) [10 points] The size $P(t)$ of a certain population satisfies

$$\frac{dP(t)}{dt} = (0.2)\, P(t).$$

If $P(0) = 1000$, approximately when is $P(t) = 2718$.

b) [10 points] Find the equation of the tangent line, at $x = 1/4$, to the curve

$$y = \arctan(4x)$$

7. [20 points] A particle is moving along the x-axis at a constant speed $dx/dt = 25$. How fast is its distance to the point $P(0,3)$ changing when $x = 4$. How fast is the angle θ changing when $x = 4$.

8. [20 points] A rectangular box has a square base and no top. If its surface area is equal to 48, find the dimensions of the box that maximize its volume. Verify that the answer you obtained is a local maximum.