

MATH 221

SECOND EXAM

PASSMAN

NAME _____

T. A.'s NAME _____

**Do all 6 problems and show all work.
Otherwise full credit will not be given.
Only use techniques already covered in class.**

PROBLEM	GRADE
1 20 pts	
2 20 pts	
3 20 pts	
4 20 pts	
5 20 pts	
6 20 pts	
TOTAL	

1. a) (10 pts) Find the slope of the curve

$$x^2 - xy + y^2 = 21$$

at the point $(5, 1)$.

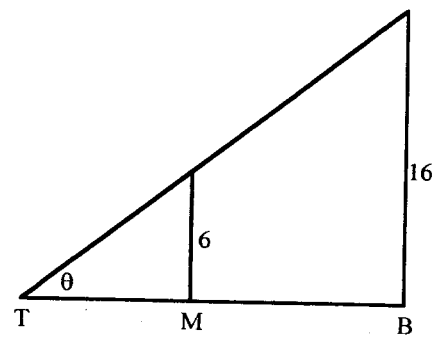
b) (10 pts) If

$$y' = \frac{y}{x} + 3,$$

find the second derivative y'' in terms of x and y . Simplify your answer.

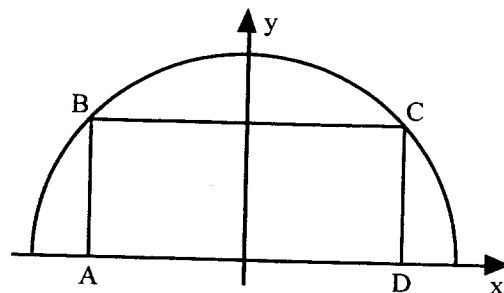
2. A man 6 feet tall walks at a rate of 5 ft/sec towards a street lamp 16 feet above the ground.

a) (10 pts) At what rate is the length of the shadow TM changing. At what rate is the tip of the shadow T moving.



b) (10 pts) How fast is the angle θ changing when the man M is 10 feet from the base B of the lamp.

3. (20 pts) Find the maximum possible perimeter of all rectangles $ABCD$ inscribed in the semicircle $y = \sqrt{5 - x^2}$ as indicated. (Use $\sqrt{5} \approx 2.25$.)

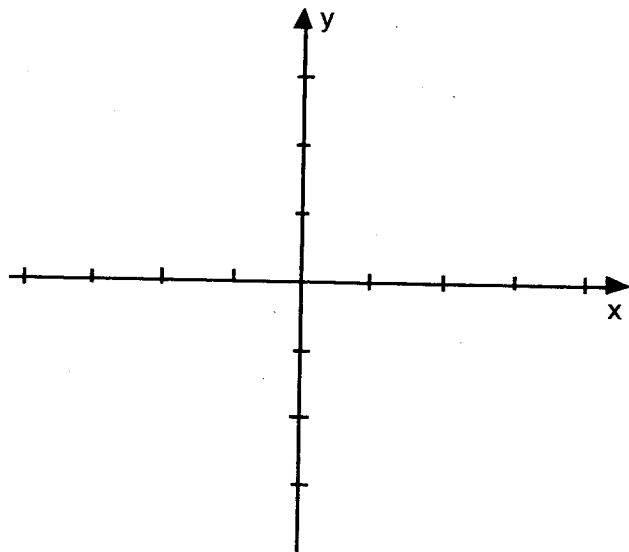


4. Let $y = x^4 - 4x^3 = x^3(x - 4)$.

a) (7 pts) Determine where the graph of y is increasing and where it is decreasing. Find the local maxima or minima, if any.

b) (7 pts) Determine where the graph of y is concave up and where it is concave down. Find the inflection points, if any.

c) (6 pts) Sketch the curve on the given coordinate system.



5. a) (10 pts) Find the point (or points) $x = c$ guaranteed by the Mean Value Theorem for the function $f(x) = x^3 - 6x^2 + 36$ in the interval $[1, 4]$.

b) (10 pts) Determine the indefinite integral

$$\int (x + 1) \cdot \sqrt{x^2 + 2x + 3} \, dx.$$

6. a) (10 pts) Find the solution of the differential equation $y' = -2xy^2$ that goes through the point $(0, 1)$.

b) (10 pts) Suppose that $\sum_{i=1}^n a_i = 4$ and $\sum_{i=1}^n b_i = 9$. Determine

$$\sum_{i=1}^n (2^{i+1} - 2^i + 5a_i - 2b_i).$$

